

On Methods of Searching for Generalized Solutions of Simple Differential Equations

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Abstract. The article discusses methods for solving simple differential equations of generalized functions.

Keywords: generalized function, differential equations, simple, solution, example

Examples of finding generalized solutions of simple differential equations using classical solutions are given.

Suppose we have a simple m -order differential equation

$$\sum_{k=0}^m a_k(x)y^{(k)} = f(x) \quad (1)$$

Here $a_k(x) \in C^{(\infty)}(R^1)$ va $f \in D'(R^1)$.

Definition: Optional $\varphi(x)$ for $\in D(R^1)$ in the generalized sense of equation (1), i.e.

$$\left(\sum_{k=0}^m a_k(x)y^{(k)}, \varphi \right) = (f, \varphi)$$

A generalized function $y(x) \in D'(R^1)$ satisfying the equation is called a generalized solution to equation (1).

Consider examples of finding generalized solutions to simple differential equations.

1-Example. Find a generalized general solution of the equation $y'=0$ in the space $D'(R^1)$?

Solution. Suppose there is a solution $y \in D'$. Anyway in this case The following equation is valid for a principal function $\varphi \in D$.

$$(y', \varphi) = 0 \quad (2)$$

It is known that for an arbitrary function $\varphi_0(x)$ satisfying the condition $\int_{-\infty}^{+\infty} \varphi_0(x)dx = 1$, an arbitrary function $\varphi \in D(R^1)$ can be expressed as follows:

$$\varphi(x) = \varphi_0 \int_{-\infty}^{+\infty} \varphi(x)dx + \varphi_1'(x), \varphi_1 \in D(R^1), \quad (3)$$

Considering (3), we can write the following:

$$\begin{aligned} (y, \varphi) &= \left(y, \varphi_0 \int_{-\infty}^{+\infty} \varphi(x)dx + \varphi_1'(x) \right) = \\ &= (y, \varphi_0) \int_{-\infty}^{+\infty} \varphi dx + (y, \varphi_1') \end{aligned} \quad (4)$$

From here we take (2) and form $(y, \varphi_1') = 0$ va $(y, \varphi_0) = c$. In this case

$$(y, \varphi) = c \int_{-\infty}^{+\infty} \varphi dx = (c, \varphi), \quad \forall \varphi \in D$$

that is, we create $y = c$.

2-Example. $y^{(m)} = 0, m = 2, 3, \dots$

Solution. The equation $y^{(m-1)} = z, y^{(m-2)} = z, \dots$ can be reduced to solving a simple differential equation of the form $z' = f(x)$.

Using the result of Example 1, we see that the general solution of a simple differential equation of order m has the following form:

$$y(x) = c_0 + c_1 + \dots + c_{m-1}x^{m-1}.$$

Now let's look at simple differential equations with variable coefficients:

1. $xy' = 1$; 2. $x^2y' = 0$; 3. $y'' = \delta(x)$
4. $(x+1)y'' = 0$; 5. $(x+1)^2y'' = 0$; 6. $(x+1)y''' = 0$

To solve these equations, we use $\theta(x)$ - in the cavity and $\delta(x)$ –Dirac functions δ and their derivatives; It is known that the equation $\theta'(x) = \delta(x)$ is true.

According to the definition of a generalized solution and the rules for calculating generalized products, generalized solutions of the above equations have the following form.

1. $y(x) = c_0 + c_1\theta(x) + \ln|x|$
2. $y(x) = c_0 + c_1\theta(x) + c_3\delta(x)$
3. $y(x) = c_0 + c_1x + x\theta(x)$
4. $y(x) = c_0 + c_1(x) + c_2\theta(x+1)(x+1)$
5. $y(x) = c_0 + c_1(x) + c_2(x+1) + c_3\theta(x+1)(x+1)$
6. $y(x) = c_0 + c_1(x) + c_2x^2 + c_3\theta(x+1)(x+1)^2$

We present an equation for finding generalized general solutions of homogeneous simple differential equations with a constant second-order coefficient and their solution:

$$af''(x) + bf'(x) + cf(x) = m\delta(x) + n\delta'(x)$$

Example 1. $f''(x) + 2f'(x) + f(x) = 2\delta(x) + \delta'(x)$

Solution. $f(x) = \theta(x)e^{-x}(1+x)$.

Example 2. $f''(x) + 4f(x) = \delta(x)$

Solution. $f(x) = \frac{1}{2}\theta(x)\sin 2x$.

Example 3. $f''(x) - 4f(x) = \delta(x) + \delta'(x)$

Solution. $f(x) = \theta(x)e^{2x}$.

The solutions to these equations are found by searching in the form $f(x) = \theta(x)z(x)$, $z \in C^2(R')$.

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