IJIAET International Journal of Innovative Analyses and Emerging Technology

| eISSN: 27924025 | http://openaccessjournals.eu | Volume: 1 Issue: 4

Free Vibrations of an Elastic Two-Layer Plate

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ABSTRACT: The theory of free vibrations of a two-layer elastic plate is developed on the basis of a flat formulation of the problem on the basis of exact solutions of the equations of the linear theory of elasticity in transformations. Equations of vibrations of symmetric vibrations of an infinite in plan two-layer plate are obtained. An algorithm is proposed that allows one to uniquely determine the VAT of an arbitrary layer of a plate.

Keywords: two-layer and three-layer plate, vibrations, displacements, stresses.

Introduction

The theory of elastic plates is one of the sections of the three-dimensional theory of elasticity. In this section, we consider such problems of their calculation, under which the boundary conditions on the lateral surfaces of the plate are specified in stresses. In this case, the construction of the basic relations of the theory of plates consists in reducing the three-dimensional problem to the two-dimensional one. To achieve the goal, various methods and approaches are used. Usually, various simplifying hypotheses and prerequisites are used for this. These hypotheses and prerequisites, together with simplifications, lead to significant shortcomings and errors [1].

In recent decades, in various fields of technology and construction, multilayer, in particular, three-layer plates have been widely used. In this case, in many cases, dynamic calculations of plates are carried out according to the classical Kirchhoff theory. Therefore, very often, such calculations turn out to be suitable only for low-frequency oscillatory processes. These include a considerable amount of research. Some of them are analyzed in [2,3,4].

These disadvantages apply to both classical and refined theories of plate vibration. Therefore, many researchers have attempted to refine the differential equations of oscillation [5,6]. At the same time, they try to derive the equations of oscillations that take into account certain factors of a physical, mechanical or geometric nature. Further development and refinement of the classical theory can be divided into two directions: the development of asymptotic theory and theories of the Timoshenko and Reissner type [7]. In addition, depending on the factors taken into account, the methods for deriving the differential equations of vibration, based on the dynamic theory of elasticity, are divided into other directions.

One of them is the method of using general solutions of three-dimensional problems of the dynamic theory of elasticity, which was developed in [4]. An essential and successful application to problems of dynamics, this method was obtained in the works of IG Filippov and his students [8,9]. It is based on the use of integral transformations in coordinate and time. It effectively uses general solutions of three-dimensional problems of the theory of elasticity (viscoelasticity) in transformations. Subsequently, these solutions are expanded in power series for the approximate satisfaction of the dynamic conditions specified on the boundary surfaces of the plate [10, 11].

The essence of the method is reduced to the study of the constructed solutions for various types of external influences. Elucidation of the conditions under which the displacements or their "main parts" satisfy simple differential equations in partial derivatives forms the basis of the method [12]. This includes the creation of an algorithm that allows the field of these "main parts" to calculate the approximate values of the fields of displacements and stresses.

Problem statement and solution method. In this article, a two-layer plate is considered in the Cartesian coordinate system. Thus, the Ox axis is directed along the centerline of the longitudinal section, and the Oz axis is directed vertically upward.

Let's enumerate the layers of the plate as the top layer is called the first layer, the bottom layer is called the second layer.Let h_1 and h_2 be the thicknesses of the first and second layers, respectively; λ_k, μ_k, are the elastic constants of the materials of the layers, i.e. Lamé coefficients; ρ_k -bulk density of layers $(k=1,2)$. The plate, we will consider, is hingedly supported in the longitudinal direction along two e and e edges. As the governing equations, we take the equations of oscillation in dimensionless variables [6].

Dependences of stresses $\sigma_{ij}^{(k)}$ on deformations $\varepsilon_{ij}^{(k)}$ at the points of the layers of the plate are described by Hooke's law for each layer $(k=1,2)$. Equations of motion of the points of the constituent layers in the Cartesian coordinate system

$$\sigma_{ij,j}^{(k)} = \rho_k \ddot{U}_i^{(k)} \qquad (1)$$

are greatly simplified by introducing the potentials φ_k and $\vec{\psi}_k$ of longitudinal and transverse waves according to the formula

$$\vec{U}^{(k)} = grad\varphi_k + rot\vec{\psi}_k \qquad (2)$$

and take the form of wave equations.

$$\lambda_{k1}(\Delta\phi_k) = \rho_k\ddot{\phi}_k; \quad \mu_k(\Delta\vec{\psi}_k) = \rho_k\ddot{\vec{\psi}}_k, \qquad (3)$$

where - Δ is the differential Laplace operator;

In the case of plane deformation, taking into account that the vectors of displacements of the points of the layers are equal

$$\vec{U}^k = U_k \cdot \vec{i} + W_k \cdot \vec{k}, \quad U_k = U_k(x,z,t); \quad W_k = W_k(x,z,t), \qquad (4)$$

where $\vec{i}$, $\vec{k}$ are the unit vectors of the coordinate axes, it suffices to set

$$\varphi_k = \varphi_k(x,z,t); \quad \vec{\psi}_k = \psi_k(x,z,t)\vec{j}, \qquad (5)$$

where - $\vec{j}$ is the unit unit vector of the axis so that the equations of motion of the points of the layers of the plate take the form

$$(\lambda_k + 2\mu_k)(\Delta\phi_k) = \rho_k\frac{\partial^2\phi_k}{\partial t^2}; \quad \mu_k(\Delta\psi_k) = \rho_k\frac{\partial^2\psi_k}{\partial t^2}, \qquad (6)$$

$$\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2.$$

By virtue of the Helmholtz theorem, in the absence of internal sources, the vector potentials $\vec{\psi}_m$ of transverse waves must satisfy the conditions for the solenoidal nature of the vector fields

$$div\vec{\psi}_k = 0, \quad k=1,2,$$

which in case (2) are performed automatically.

It is assumed that at $t<0$ the plate was at rest, and at moment $t=0$ dynamic actions are applied to its boundary surfaces

at $z = \frac{h_2}{2} + h_1$: $\quad \sigma_{xz}^{(i)}(x,z,t) = F_x^{(i)}(x,t);\ \sigma_{zz}^{(i)}(x,z,t) = F_z^{(i)}(x,t);\ \sigma_{yz}^{(i)}(x,z,t) = 0, \qquad (7)$

at $z = -\frac{h_2}{2}$: $\quad \sigma_{xz}^{(i)}(x,z,t) = -F_x^{(i)}(x,t);\ \sigma_{zz}^{(i)}(x,z,t) = -F_z^{(i)}(x,t);\ \sigma_{yz}^{(i)}(x,z,t) = 0. \qquad (8)$

In addition, on the surfaces with the second layer $z = \frac{h_2}{2}$ there are dynamic and kinematic contact conditions

ISSN 27924025 (online), Published under Volume: 1 Issue: 4 in September2021

IJIAET International Journal of Innovative Analyses and Emerging Technology

| eISSN: 27924025 | http://openaccessjournals.eu | Volume: 1 Issue: 4

$$\sigma_{zz}^{(1)}(x,z,t)=\sigma_{zz}^{(2)}(x,z,t);\ \sigma_{xz}^{(1)}(x,z,t)=\sigma_{xz}^{(2)}(x,z,t);\ \sigma_{yz}^{(0)}(x,z,t)=0, \quad (9)$$

$$U_1(x,z,t)\big|_{z=\frac{h_2}{2}}=U_2(x,z,t)\big|_{z=\frac{h_2}{2}};\ W_1(x,z,t)\big|_{z=\frac{h_2}{2}}=W_2(x,z,t)\big|_{z=\frac{h_2}{2}}. \quad (10)$$

The initial conditions of the problem are assumed to be zero, i.e. at $t=0$

$$\varphi_k=\psi_k=0,\ \frac{\partial\varphi_k}{\partial t}=\frac{\partial\psi_k}{\partial t}=0. \quad (11)$$

This, in the general formulation of the problem of unsteady vibrations of the plate, the problem is reduced to the solution for each layer of two equations - (6), with boundary (7), (8) and contact - (9), (10) and zero initial conditions - (eleven).

To solve the problem, we will choose the potential functions ψ_k and φ_k in the form of the given work [4]. We will also describe the displacement of the point of the layers of a two-layer plate as in [4]. After that, these expressions of displacements, equating to expressions of potential functions, we arrive at new unknown functions to be determined. Using the contact conditions, we obtain a system representing the general equations of oscillation. From the resulting system, we obtain the following system of equations for symmetric vibrations of a two-layer plate.

$$\left\{A_{11}\frac{\partial^4}{\partial t^4}+A_{12}\frac{\partial^4}{\partial x^2\partial t^2}+A_{13}\frac{\partial^4}{\partial x^4}+A_{14}\frac{\partial^2}{\partial t^2}+A_{15}\frac{\partial^2}{\partial x^2}+A_{16}\right\}\frac{\partial}{\partial x}W_0^{(0)}+$$

$$+\left\{B_{11}\frac{\partial^4}{\partial t^4}+B_{12}\frac{\partial^4}{\partial x^2\partial t^2}+B_{13}\frac{\partial^4}{\partial x^4}+B_{14}\frac{\partial^2}{\partial t^2}+B_{15}\frac{\partial^2}{\partial x^2}\right\}U_0^{(0)}= \quad (12)$$

$$=\left\{C_{11}\frac{\partial^4}{\partial t^4}+C_{12}\frac{\partial^4}{\partial x^2\partial t^2}+C_{13}\frac{\partial^4}{\partial x^4}+C_{14}\frac{\partial^2}{\partial t^2}+C_{15}\frac{\partial^2}{\partial x^2}+C_{16}\right\}f_x^{(1)}(k,p);$$

$$\left\{A_{21}\frac{\partial^2}{\partial t^2}+A_{22}\frac{\partial^2}{\partial x^2}+A_{23}\right\}W_0^{(0)}+\left\{B_{21}\frac{\partial^2}{\partial t^2}+B_{22}\frac{\partial^2}{\partial x^2}+B_{23}\right\}\frac{\partial U_0^{(0)}}{\partial x}=C_{21}f_z^{(1)}(k,p).$$

Here, $W_0^{(0)}$ and $U_0^{(0)}$ are the sought functions, A_{ij}, B_{ij}, C_{ij} $(i,j=1,2)$ -are constants associated with the elastic characteristics of the layers and their sizes. For example,

$$A_{11}=-\left(\frac{q_0}{a_1^2b_1^2}+\frac{1-q_1}{a_0^2a_1^2}\right)\frac{z_1h_0^4}{12}-\left(\frac{2q_1}{a_1^2b_0^2}+\frac{3(1+q_1-3q_0q_1)}{a_1^2b_1^2}+\frac{2q_0q_1}{a_0^2a_1^2}+\frac{3q_0(1-q_1)}{a_1^4}+\frac{1+q_1}{b_0^2b_1^2}+\frac{q_0(1+q_1)}{a_0^2b_1^2}\right)\frac{z_1^3h_0^2}{36} \text{ etc.}$$

The system of equations for the vibration of a two-layer plate (12) is reduced to a dimensionless form. For this, we introduce the following dimensionless quantities.

$$x^*=\frac{x}{l},\ z^*=\frac{z}{l},\ h_m^*=\frac{h_m}{l},\ t^*=\frac{t\cdot b_0}{l},\ a_m^*=\frac{a_m}{b_0},\ b_m^*=\frac{b_m}{b_0},\ \mu_m^*=\frac{\mu_m}{\mu_0},\ U_0^{(0)*}=\frac{U_0^{(0)}}{l},\ W_0^{(0)*}=\frac{W_0^{(0)}}{l},\ \xi^*=\frac{\xi}{l}. \quad (13)$$

Introducing dimensionless quantities (13) into (12), then performing some mathematical transformations, we obtain the following system of equations

$$\left\{A^*_{11}\frac{\partial^4}{\partial t^{*4}}+A^*_{12}\frac{\partial^4}{\partial x^{*2}\partial t^{*2}}+A^*_{13}\frac{\partial^4}{\partial x^{*4}}+A^*_{14}\frac{\partial^2}{\partial t^{*2}}+A^*_{15}\frac{\partial^2}{\partial x^{*2}}+A^*_{16}\right\}\frac{\partial}{\partial x^*}W_0^{*(0)}+$$

ISSN 27924025 (online), Published under Volume: 1 Issue: 4 in September2021

IJIAET International Journal of Innovative Analyses and Emerging Technology
| eISSN: 27924025 | http://openaccessjournals.eu | Volume: 1 Issue: 4

$$+\left\{B^*_{11}\frac{\partial^4}{\partial t^{*4}}+B^*_{12}\frac{\partial^4}{\partial x^{*2}\partial t^{*2}}+B^*_{13}\frac{\partial^4}{\partial x^{*4}}+B^*_{14}\frac{\partial^2}{\partial t^{*2}}+B^*_{15}\frac{\partial^2}{\partial x^{*2}}\right\}U_0^{*(0)}= \quad (14)$$

$$=\left\{C^*_{11}\frac{\partial^4}{\partial t^{*4}}+C^*_{12}\frac{\partial^4}{\partial x^{*2}\partial t^{*2}}+C^*_{13}\frac{\partial^4}{\partial x^{*4}}+C^*_{14}\frac{\partial^2}{\partial t^{*2}}+C^*_{15}\frac{\partial^2}{\partial x^{*2}}+C^*_{16}\right\}f_x^{*(1)}(k,p);$$

$$\left\{A^*_{21}\frac{\partial^2}{\partial t^{*2}}+A^*_{22}\frac{\partial^2}{\partial x^{*2}}+A^*_{23}\right\}W_0^{*(0)}+\left\{B^*_{21}\frac{\partial^2}{\partial t^{*2}}+B^*_{22}\frac{\partial^2}{\partial x^{*2}}+B^*_{23}\right\}\frac{\partial U_0^{*(0)}}{\partial x}=C^*_{21}f_z^{*(1)}(k,p)$$

Here

$$A^*_{11}=-\left(\frac{q_0}{a_1^{*2}b_1^{*2}}+\frac{1-q_1}{a_0^{*2}a_1^{*2}}\right)\frac{z_1^*h_0^{*4}}{12}-\left(\frac{2q_1}{a_1^{*2}b_0^{*2}}+\frac{3(1+q_1-3q_0q_1)}{a_1^{*2}b_1^{*2}}+\frac{2q_0q_1}{a_0^{*2}a_1^{*2}}+\right.$$

$$\left.+\frac{3q_0(1-q_1)}{a_1^{*4}}+\frac{1+q_1}{b_0^{*2}b_1^{*2}}+\frac{q_0(1+q_1)}{a_0^{*2}b_1^{*2}}\right)\frac{z_1^{*3}h_0^{*2}}{36} \text{ etc.}$$

Further, on the basis of eqs. (14), we solve the problem of symmetric vibrations of a two-layer plate, the size along the axis, which is equal to l. For x = 0 and x = l, i.e. the boundary conditions of the problem, in accordance with the conditions for fixing the edges of the plate [7-10], have the form:

$$\frac{\partial U_0^{*(0)}(x,t)}{\partial x}=0;\quad \frac{\partial^3 U_0^{*(0)}(x,t)}{\partial x^3}=0;$$
$$W_0^{*(0)}(x,t)=0;\quad \frac{\partial^2 W_0^{*(0)}(x,t)}{\partial x^2}=0. \quad (15)$$

Solutions to the system of equations (14) are sought in the form of a sum of harmonic solutions along the coordinate, i.e. as sums of trigonometric functions. Such solutions, satisfying the boundary conditions (15), will be

$$W_0^{*(0)}=W_1(t)\sin\frac{\pi x}{l},\ U_0^{(0)}=U_1(t)\cos\frac{\pi x}{l}. \quad (16)$$

In this case, the functions $f_x(x,t)$ and $f_z(x,t)$ should also be represented as

$$f_x=f_{x1}(t)\cos\frac{\pi x}{l};\qquad f_z=f_{z1}(t)\sin\frac{\pi x}{l}. \quad (17)$$

Further, substituting (16) and (17) into the system of equations (14), we obtained the system of equations for the functions $U_m(t)$ and $W_m(t)$

$$\left\{T_{11}\frac{\partial^4}{\partial t^4}+T_{12}\frac{\partial^2}{\partial t^2}+T_{13}\right\}W_1(t)+\left\{N_{11}\frac{\partial^4}{\partial t^4}+N_{12}\frac{\partial^2}{\partial t^2}+N_{13}\right\}U_1(t)=H_{11};$$
$$\left\{T_{21}\frac{\partial^2}{\partial t^2}+T_{22}\right\}W_1(t)+\left\{N_{21}\frac{\partial^2}{\partial t^2}+N_{22}\right\}U_1(t)=H_{21}. \quad (18)$$

where $T_{11}=\tilde{A}_{11}\frac{\pi}{l}$; $T_{12}=\tilde{A}_{14}\frac{\pi}{l}-\tilde{A}_{12}\left(\frac{\pi}{l}\right)^3$; … etc.

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| eISSN: 27924025 | http://openaccessjournals.eu | Volume: 1 Issue: 4

$$H_{11}=\left[C_{11}\frac{\partial^4}{\partial t^4}+\left(C_{14}-C_{12}\left(\frac{\pi}{l}\right)^2\right)\frac{\partial^2}{\partial t^2}+C_{13}\left(\frac{\pi}{l}\right)^4-C_{15}\left(\frac{\pi}{l}\right)^2+C_{16}\right]f_{1x}(t),\ H_{21}=C_{21}f_{1z}(t)\ ,.$$

The initial conditions for the functions $U_1(t)$ and $W_1(t)$ are of the form

$$U_1(x,t)=\frac{\partial U_1(x,t)}{\partial t}=\frac{\partial^2 U_1(x,t)}{\partial t^2}=\frac{\partial^3 U_1(x,t)}{\partial t^3}=0;$$
$$W_1(x,t)=\frac{\partial W_1(x,t)}{\partial t}=\frac{\partial^2 W_1(x,t)}{\partial t^2}=\frac{\partial^3 W_1(x,t)}{\partial t^3}=0. \quad (19)$$

Thus, the original problem is reduced to integrating a system of two ordinary differential equations (18) with initial conditions (19).

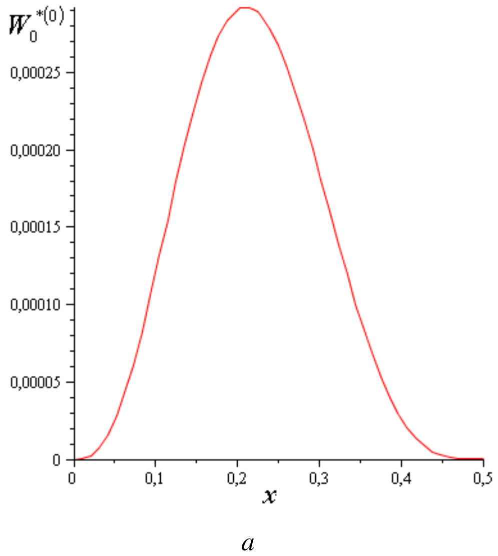
Calculation results. The system of equations was solved numerically using the Maple-17 software package. For the calculations, the following values of the geometrical and physical-mechanical characteristics of the layers were taken:

$l=0.5m$; $h_1=0.004m$; $h_2=0.05m$; $\xi=0.3h_0$; $\nu_1=\nu_2=0.33$; $\rho_1=\rho_2=2700kg/m^3$; $E_1=E_2=69\cdot10^9\ Pa$; $f_x^{(1)}=f_z^{(2)}=50\ N$.

Based on the results of solving the system of differential equations (14), using formulas (16), the main parts of the transverse - $W_0^{*(0)}$ and longitudinal - $U_0^{*(0)}$ longitudinal displacements of the points of the plane $\xi=0.3h_0$ of the middle layer depending on the coordinate are calculated. The results are shown in Fig. 1a, b.

Then the displacements are expressed in terms of their own principal parts $W_0^{*(0)}$ and $U_0^{*(0)}$, which are the sought functions of the system of equations (14), according to the following formulas:

$$W_0^*=\frac{1}{\xi^*}\left(1+\frac{q_0}{a_0^{*2}}\right)\frac{z^{*3}}{6}\frac{\partial^2 W_0^{(0)*}}{\partial t^{*2}}-(1+q_0)\frac{z^{*3}}{6}\frac{1}{\xi^*}\frac{\partial^2 W_0^{(0)*}}{\partial x^{*2}}+\frac{1}{\xi^*}z^*W_0^{(0)*}+ \quad (20)$$
$$+\frac{q_0}{a_0^{*2}}\frac{z^{*3}}{6}\frac{\partial^3 U_0^{(0)*}}{\partial t^{*2}\partial x^*}-q_0\frac{z^{*3}}{6}\frac{\partial^3 U_0^{(0)*}}{\partial x^{*3}}.$$



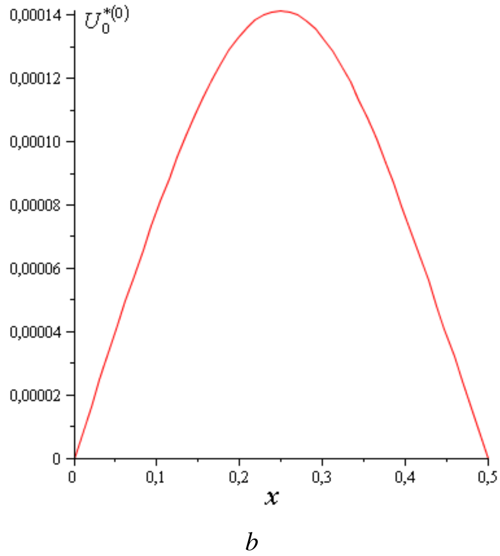
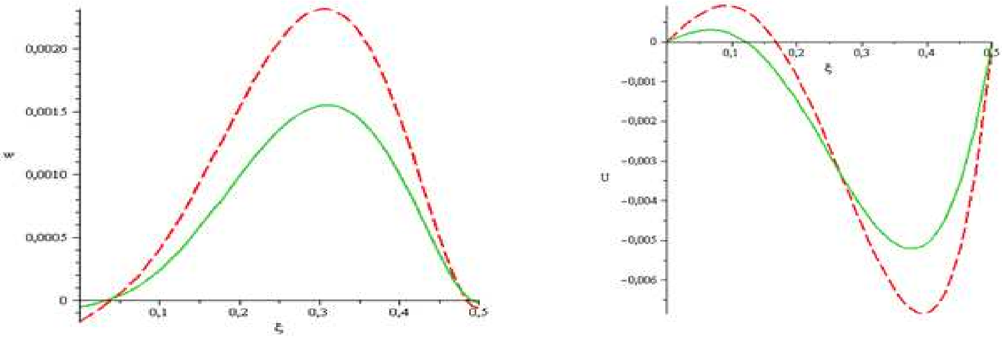


Fig. 1. Graphs of functions $W_0^{*(0)}$ (a) and $U_0^{*(0)}$ (b).

These formulas make it possible to calculate displacements based on the results of solving the system of oscillation equations (14). The results obtained are shown in Fig. 2, from which it follows that the nature of the change in displacements is sinusoidal.



a *b*

Fig. 2. Graphs of transverse W_0^* (a) and longitudinal U_0^* (b). displacements of points of the plate at - $z_1^* = h/2$ dashed line; - $z_1^* = h/3$ solid line.

In this case, the values of the longitudinal displacement are an order of magnitude higher than the corresponding values of the lateral displacement. This result fully corresponds to the physical essence of the problem, since symmetric (longitudinal) vibrations of the plate are considered.

Conclusions. The theory of unsteady symmetric vibrations of a two-layer elastic plate in a flat formulation, free from hypotheses and preconditions, has been developed. In a particular case, when the plate is homogeneous, the equations of oscillation (12) transform into the well-known [9-10] equations of oscillation of a single-layer elastic plate;

With symmetric vibrations of a two-layer plate, the appearance of insignificant transverse displacements is caused by the action of longitudinal external loads $f_x^{(1)}$ on the front and back sides of the plate. These displacements are insignificant and an order of magnitude less in comparison with longitudinal displacements. Therefore, the lateral displacements of the points of the layers can be neglected.

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ISSN 27924025 (online), Published under Volume: 1 Issue: 4 in September2021