

The Economic Meaning of Dual Issues

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Abstract: Any linear programming problem is inextricably linked to another problem called a dual problem. The connection between problems is such that the solution of any one of them can be determined using the solution of the other.

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Any linear programming problem is inextricably linked to another problem called a dual problem. The connection between problems is such that the solution of any one of them can be determined using the solution of the other. We call such interrelated issues collectively dual issues.

As an example, let's consider the issue of production planning. Let the company produce different products. For the production of this product, the enterprise should have m different production means $b_i (i = \overline{1, m})$ in quantities. Let the amount of j -means ($j = \overline{1, n}$) used for the production of one unit of each different product be one unit. It is necessary to plan production in such a way that, as a result, maximum output in terms of money is produced (c_y) using limited resources.

We define x_j - the quantity of different products x_j to be produced. Then the mathematical model of the problem will look like this:

$$(1) \begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2 \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m \end{cases}$$

$$(2) x_j \geq 0, (j = \overline{1, n})$$

$$(3) Y_{\max} = c_1x_1 + c_2x_2 + \dots + c_n x_n$$

Now let's evaluate the means of production. We assume that the cost of inputs and the cost of output have the same unit of measurement. with $\omega_i (i = \overline{1, m})$ determine the price of one unit of the same tool. In that case, the price of the means of production used for the production of all kinds of products

$$\sum_{i=1}^n a_n \omega_i$$

constitutes a unit. The cost of all the tools used should not exceed the cost of the manufactured product, that is:

$$\sum_{i=1}^n a_n \omega_i \geq C_j, (j = 1, 2 \dots n)$$

The price of all available tools

$$\sum_{j=1}^m b_j \omega_j$$

expressed through

Thus, the mathematical model of the given problem (1)-(2) will have the following form:

$$(4) \begin{cases} a_{11}\omega_1 + a_{12}\omega_2 + \dots + a_{1n}\omega_n \leq c_1 \\ a_{21}\omega_1 + a_{22}\omega_2 + \dots + a_{2n}\omega_n \leq c_2 \\ a_{m1}\omega_1 + a_{m2}\omega_2 + \dots + a_{mn}\omega_n \leq c_m \end{cases}$$

$$(5) Z_{\min} = b_1\omega_1 + b_2\omega_2 + \dots + b_m\omega_m$$

The given issue and its dual $b, (i = \overline{1, m})$ issue can be interpreted as follows from an economic point of view:

Given problem: using limited resources, how much of which product is produced (given the price of the product $(c_j, (j = \overline{1, n}))$) will the monetary expression of all products produced be the maximum?

Dichotomous problem: given the price $b, (i = \overline{1, m})$ of a unit of production using limited means, what should be the price of each unit of means so that the monetary expression of the total cost is minimal?

The variables in the dichotomous problem ω_i are called the values of the instrument. It seems that there is a connection between mathematical models of given and dual problems. The matrix A consisting of the coefficients of the given problem is the transposed matrix in the dual problem, the coefficients of the linear function c_j in the given problem consist of the free terms in the given problem, the free terms in the conditions of the given problem consist of the coefficients of the linear function of the dual problem.

Depending on how the problems are presented, they are divided into symmetric and asymmetric dual problems.

Non-symmetric dual problems

In non-symmetric dual problems, the boundary conditions in the given problem are from the equations, the boundary conditions in the dual problem

and conditions consist of inequalities. For example, the matrix expression of non-symmetric dual problems is:

Given problem:

$$(1) AX = b$$

$$(2) X \geq 0$$

$$(3) Y_{\min} = CX$$

That is, it is necessary to find such a vector satisfying the conditions (1) and (2) that gives the minimum $x = (x_1, x_2, \dots, x_n$ value to the linear function (3).

Dual issue:

$$(4) WA \leq C$$

$$(5) Z_{\min} = WB$$

that is, it is necessary to find such a vector series satisfying the conditions (4) $W = (\omega_1, \dots, \omega_m)$ that gives the maximum value to the linear function $C = (C_1, C_2, \dots, C_n)$ (5). In both problems, the vector is the $b = (b_1, b_2, \dots, b_m)$ row, the vector is the column $A = (a_n)$, and the matrix consists of the coefficients of the boundary conditions. The optimal solutions of these problems are connected based on the following theorem.

Theorem. If one of the given problem or its dual problem has an optimal solution, then the other one will also have a solution. And the extreme values of the linear functions in these problems are mutually equal, i.e

$$(6) Y_{\min} = Z_{\max}$$

If the linear function of one of these problems is unbounded, then the second problem also has no solution.

Symmetric dual problems.

The difference between symmetric dual problems and non-symmetric dual problems is that the boundary conditions in the given and dual problems consist of inequalities, and the unknowns in the dual problem are subject to non-negativity.

Given problem:

$$(1) AX \geq b$$

$$(2) X \geq 0$$

$$(3) Y_{\min} = CX$$

It is necessary to find such a vector column $x = (x_1, x_2, \dots, x_n)$ satisfying the conditions (1) and (2) that gives a minimum value to the linear function (3).

A dilemma

$$(4) WX \leq C$$

$$(5) W \geq 0$$

$$(6) Z_{\min} = Wb$$

It is necessary to find a vector that satisfies the conditions (4) and (5) and gives the maximum value to the linear function $W = (\omega_1, \dots, \omega_m)$ (6).

A system of inequalities can be transformed into a system of equations with the help of additional variables. Therefore, symmetric dual problems can be transformed into non-symmetric dual problems. So, the theorem about the solutions of non-symmetric dual problems is also valid for symmetric dual problems.

An example. The following problem is a double problem.

The conditions of the problem consist of inequalities, so it is necessary to create a dual problem that is symmetric to the given problem. For this, it is necessary to reduce the given problem to the 3rd form, to achieve this, inequality 1 is multiplied by 1. As a result, the following symmetric dual problems are formed: dual problem:

$$\begin{cases} x_1 - x_2 - x_3 \leq 4 \\ x_1 - 5x_2 + x_3 \geq 5 \\ 2x_1 - x_2 + 3x_3 \geq 6 \end{cases}$$

$$x_j \geq 0, j = 1, 2, 3$$

$$Y_{\min} = 2x_1 + x_2 + 5x_3$$

Given problem:

$$\begin{cases} -\omega_1 + \omega_2 + 2\omega_3 \leq 2 \\ \omega_1 - 5\omega_2 + \omega_3 \leq 1 \\ \omega_1 + \omega_2 + 3\omega_3 \leq 5 \end{cases}$$

$$\omega_i \geq 0, (i = 1, 2, 3)$$

$$Y_{\min} = 2x_1 + x_2 + 5x_3$$

Dual issue:

$$\begin{cases} -x_1 + x_2 + x_3 \geq 4 \\ x_1 - 5x_2 + x_3 \geq 5 \\ 2x_1 - x_2 + 3x_3 \geq 6 \end{cases}$$

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