Solving Cauchy Problems Using Euler Methods Using the C # Programming Language and Method Mapping

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ABSTRACT: This article discusses differential equations, namely, the Cauchy problem. Euler's methods for solving Cauchy problems are compared. A C # program for calculating Euler's methods is presented and the derived solutions are compared.

Keywords: Differential Equations. the Cauchy problem. Euler's method. improved Euler method. Program. C # program. Comparison of solutions.

Introduction

Empty we are given the following Cauchy problem and we must solve it using one of the Euler methods:

$$y' = x^2 + y^2$$
$$y(0) = 0.4, \quad x \in [0,1]$$

From the beginning we will solve this problem by the first Euler method and then by the improved Euler method. And we will compare the values of the results, we will lead to the conclusion, drawing a conclusion which of the methods actually works and which gives a little bit of zooke during the calculation.

We will first find solutions to this problem using the first Euler method. To do this, we will use the following Euler formulas:

$$y'_{i+1} = y_i + f(x_i, y_i) \cdot h$$

And we will do the second calculation using the improved Euler method. To do this, we will use the following improved Euler formulas:

1)
$$x_{i+\frac{1}{2}} = x_i + \frac{h}{2};$$

2)
$$y_{i+\frac{1}{2}} = y_i + f(x_i, y_i);$$

3)
$$y_{i+1} = y_i + f(x_{i+\frac{1}{2}}, y_{i+\frac{1}{2}});$$

Main part

We will solve everything according to the above formulas. During the calculation, we use the following expressions:

$$n = 10, \quad x_0 = 0, \quad x_{10} = 1, \quad y_0 = 0.4, \quad f(x, y) = x^2 + y^2;$$

And we will come to the following comparative results:

x	Метод Эйлера	Улучшенный метод Эйлера	xi+1/2	f(xi, yi)	y(i+1/2)	f(x(i+1/2), y(i+1/2))	yi+1
0	0,400	0,400	0,050	0,160	0,408	0,169	0,417
0,1	0,416	0,417	0,150	0,184	0,426	0,204	0,437
0,2	0,434	0,437	0,250	0,231	0,449	0,264	0,464
0,3	0,457	0,464	0,350	0,305	0,479	0,352	0,499
0,4	0,487	0,499	0,450	0,409	0,519	0,472	0,546

0,5	0,527	0,546	0,550	0,548	0,574	0,631	0,609
0,6	0,580	0,609	0,650	0,731	0,646	0,840	0,693
0,7	0,649	0,693	0,750	0,971	0,742	1,113	0,804
0,8	0,740	0,804	0,850	1,287	0,869	1,477	0,952
0,9	0,859	0,952	0,950	1,717	1,038	1,980	1,150
1	1,014	1,150	1,050	2,323	1,266	2,706	1,421

Now let's build a graph based on the above result. And let's make a comparison of them:
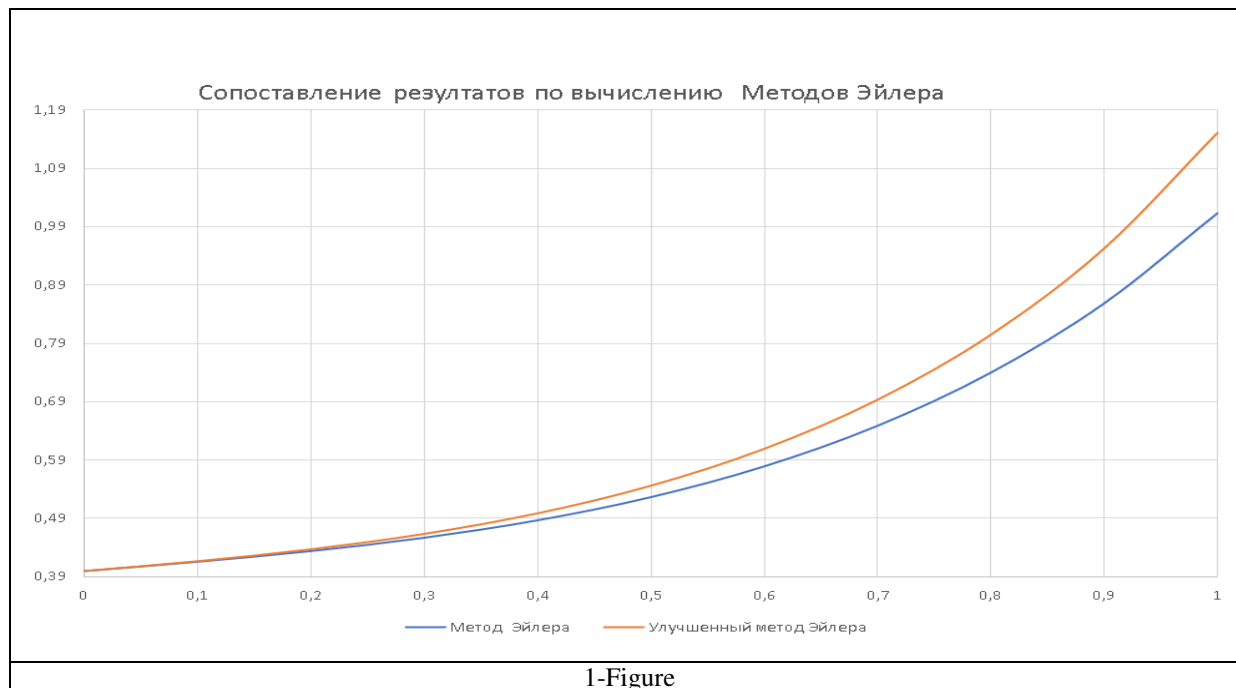
1-Figure

After comparing the results, it becomes clear that at the beginning (seen from the graph) the results overlap. As the distance from the point x0 increases, the results begin to drift apart. We can see that the more we move away from the starting point, the more significant the departure of the results. It follows from this that the second method (improved method) of Euler gives a significantly accurate result.

Now let's see another example:

$$y' = x + y\sqrt{x}$$
$$y(0) = 0.2, \ x \in [0,1]$$

And let's calculate the values using the same Euler formulas. During the calculation, we use the following expressions:

$$n = 10, \quad x_0 = 0, \quad x_{10} = 1, \quad y_0 = 0.2, \quad f(x,y) = x + y\sqrt{x};$$

And we will come to the following comparative results:

x	Метод Эйлера	Улучшенный метод Эйлера	xi+1/2	f(xi, yi)	y(i+1/2)	f(x(i+1/2), y(i+1/2))	yi+1
0	0,200	0,200	0,05	0,000	0,200	0,095	0,209
0,1	0,200	0,209	0,15	0,166	0,218	0,234	0,233
0,2	0,216	0,233	0,25	0,304	0,248	0,374	0,270
0,3	0,246	0,270	0,35	0,448	0,293	0,523	0,323
0,4	0,289	0,323	0,45	0,604	0,353	0,687	0,391
0,5	0,348	0,391	0,55	0,777	0,430	0,869	0,478
0,6	0,422	0,478	0,65	0,970	0,527	1,075	0,586
0,7	0,515	0,586	0,75	1,190	0,645	1,309	0,717
0,8	0,628	0,717	0,85	1,441	0,789	1,577	0,874
0,9	0,764	0,874	0,95	1,729	0,961	1,886	1,063
1	0,927	1,063	1,05	2,063	1,166	2,245	1,287

Now let's build a graph based on the above result. After comparing the results, it becomes clear that the second method (improved method) of Euler gives a significantly accurate result.

2-Figure
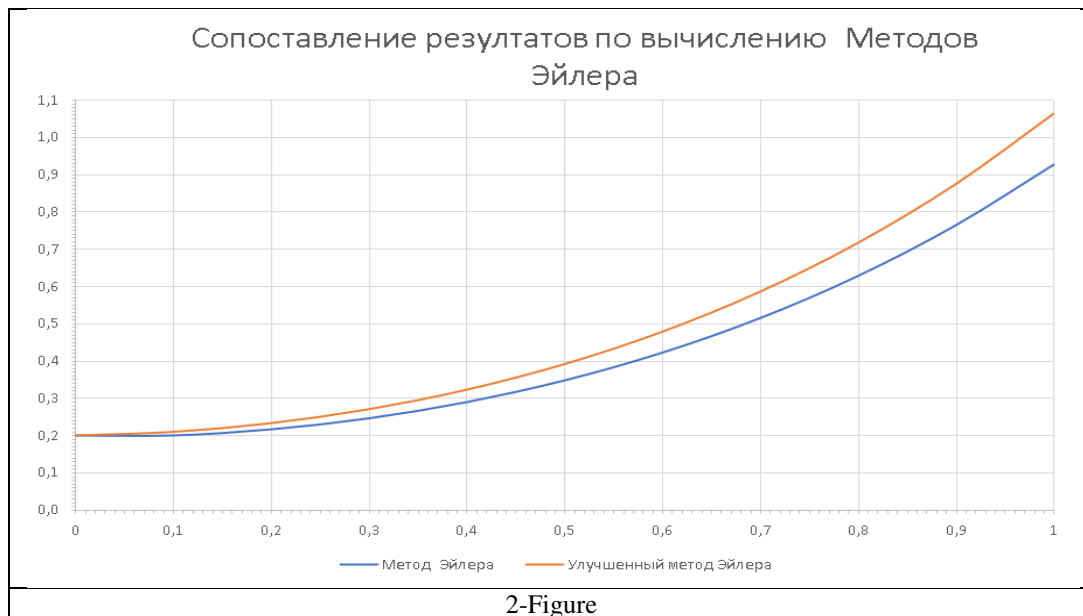
And in this example, we can see that after comparing the results, it becomes clear that at the beginning (seen from the graph) the results exactly overlap each other. As the distance from the point x0 increases, the results begin to move away from each other. We can see that the more we move away from the starting point, the more visible the difference between the results. It follows from this that the second method (improved method) of Euler gives a significantly accurate result.

Now we will carry out all these calculations using the C # programming language. The algorithm of the program will be exact as shown above. That is, the algorithm exactly converges with the manual calculation. And for this, we will only give the text of the program.

```
class Program
{
    public static void Main(string[] args)
    {
        int n;
        Console.WriteLine("n=");
        n=Convert.ToInt16( Console.ReadLine());
        double [] x=new double[n+1];
        double [] y=new double[n+1];
        double h;
        double a=0, b=1;
        y[0]=0.4;
        h=(a+b)/n;
        //Console.WriteLine(h);
        x[0]=a;
        x[10]=b;
        for(int i=1; i<=10; i++){
            x[i]=x[0]+i*h;

            y[i]=y[i-1]+f(x[i-1], y[i-1])*h;
            Console.WriteLine(y[i]);
        }
        Console.WriteLine(y[10]);
        Console.ReadKey(true);
    }

    static double f(double x, double y){
        double f=x*x+y*y;
        return f;
    }
}
```

```
C:\Users\User\Documer
n=
10
0,416
0,4343056
0,457167735419136
0,487067969249962
0,52679148991689
0,579542417301776
0,649129358646975
0,740266251072718
0,859065663320444
1,01386504471006
1,01386504471006
```

3-Figure

We see that the results with the program are identical to the manual calculation.

Так как оба примера решаются с помощью одним и тем же алгоритмом то думаю не надо приводить программу для второго примера. Он решается точно также.

Conclusion

In conclusion, we can say that after comparing the results, it becomes clear that at the beginning (seen from the graph) the results exactly fall on each other. As the distance from the point x0 increases, the results begin to move away from each other. We can see that the more we move away from the starting point, the more visible the difference between the results. It follows from this that the second method (improved method) of Euler gives a significantly accurate result. You can also say calculating by a manual method and using a programming language, we have almost identical results. That is, after the decimal point, 3 digits can be considered correct.

Литература

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