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Optimization of Neural Network Identification of a Non-Stationary Object Based On Spline Functions

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Abstract: A technique for smoothing a dynamic process based on basis-spline functions and calculating information recovery coefficients has been developed, which helps to optimize the training of a neural network data processing system by reducing the errors of the training subset. Methods and algorithms for modeling the processes of smoothing, processing, and restoring data of non-stationary processes based on cubic spline functions are studied.

Keywords: identification, non-stationary object, spline function, neural network, optimization, recognition, forecasting.

Relevance of the topic. The construction of many intelligent systems for processing information of non-stationary objects based on neural networks (NN), in particular, for solving problems of image visualization, recognition, classification of micro-objects, analysis and forecasting of technical and economic indicators, is associated with the representation, identification, and processing of images of micro-objects [1]. However, traditional approaches aimed at image visualization cannot provide the necessary accuracy in all cases, which requires the development of constructive approaches, principles, models, and algorithms, identification of dynamic processes using splines. The developed mechanisms should contribute to the efficient storage of geometric information in the numerical form with any accuracy, as well as the use of neural networks (NN) with the smallest smoothing error in adaptive learning. The mathematical apparatus of spline functions includes the calculation of generally accepted functionals, derivatives, and integrals, the solution of differential and integral equations, the construction of stable learning algorithms for neural networks, as well as the optimization of the identification of non-stationary objects [4].

It has been proven that based on splines, it is possible to develop and implement new algorithms and software tools for processing and restoring signals, which allow increasing productivity, interaction speed, and performing calculations in real-time when training NN [2,3].

The general theoretical position based on the use of local basic splines, methods and developed modified algorithms for calculating the coefficients of cubic splines that provide the smallest smoothing error are outlined below.

Mathematical model of basis splines. The construction of mathematical models for smoothing a non-stationary process using polynomial splines is associated with the solution of systems of equations that require large computational costs, and the algorithms obtained on their basis are complex [4]. The

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model of basic functions is the interpolation experimental splines $\Phi_{m,i}(x)$ [5], which are set on a Δ_x grid of the form

$$\Phi_{m,i}(x_0) = \delta_{ik} \quad (1)$$

Where δ_{ik} is the Kronecker symbol, $i = 0,1,K,n$.

The basic functions give an approximation by the Lagrange formula, as

$$f(x) \cong S_m(x) = \sum_{i=0}^{n} c_i \Phi_{m,i}(x), \quad (2)$$

Where the c_i coefficients are the values of the $f(x)$ function at the nodes of the spline.

Moreover, any polynomial spline AA of the defect degree BB can be decomposed in terms of the so-called normalized basic splines of the degree DD [6].

In this regard, $B_{m,i}(x)$, $i = -m,...,N-1$ are local piecewise polynomial functions on the segment $[a,b]$, where the mesh Δ_x:

$$\Delta_x : a = x_0 < x_1 < ... < x_{n-1} < x_n = b. \quad (3)$$

The function $B_{m,i}(x)$ as a spline of degree m of defect 1 on the grid of nodes $x_i,...,x_{i+m+1}$ is defined as

$$\frac{m}{m+1} B_{m,i}(x) = \frac{x - x_i}{x_{i+m+1} - x_i} B_{m-1,i}(x) + \frac{x_{i+m+1} - x}{x_{i+m+1} - x_i} B_{m-1,i+1}(x)$$

Note that the basic functions are transformed by means of a shift operation along the axis by an integer number of steps [7].

An NN training algorithm has been developed based on the use of the following basic functions:

1) B-splines of degree zero. B-splines, which are rectangular non-intersecting rect-impulses of length h.
2) B-splines of the first degree. B-splines, even with respect to the vertical axis and with $h = 1$ determined by the formula:

$$B_1(x) = \begin{cases} x-1, & x \in [-1,0] \\ 1-x, & x \in [0,1] \\ 0, & x \notin [-1,1] \end{cases}$$

3) B-splines of the second degree. Parabolic B-splines at $h = 1$ are determined by the expression:

$$B_2(x)=\begin{cases}0, & x\geq \frac{3}{2}\\ \frac{1}{2}\left(\frac{3}{2}-x\right)^2, & \frac{1}{2}\leq x\leq \frac{3}{2}\\ \frac{3}{4}-x^2, & 0\leq x\leq \frac{1}{2}\\ B_2(-x), & x<0\end{cases}$$

4) B-splines of the third degree. Cubic B-splines at $h=1$ are determined by the expression:

$$B_3(x)=\begin{cases}0, & x\geq 2\\ \frac{(2-x)^3}{6}, & 1\leq x<2\\ \frac{1}{6}\left(1+3(1-x)+3(1-x)^2-3(1-x)^3\right), & 0\leq x<1\\ B_3(-x), & x<0\end{cases}$$

Among them, cubic defect $d=1$ splines are the most widely common used in applications. Such splines on each of the segments $[x_i, x_{i+1}]$ coincide with cubic polynomials [8, 9].

Mechanisms for calculating the coefficients of cubic splines. Denote cubic splines by S. In practice, when smoothing with splines, the following variants of boundary conditions are usually set [10].

If the values of the first derivative of $f'(a)$ and $f'(b)$ are known at the boundary points, then it is natural to put $S_0' = f'(a)$ land $S_n' = f'(b)$. Adding these conditions leads to a system that can be solved by one of the efficient methods, such as the sweep method.

If the values of the second derivative of $f''(a)$ and $f''(b)$ are known at the ends of the interval, then they can be taken as boundary values for the values of the second derivative of the spline $S_m''(a)=S_m''(b)=0$, and the spline corresponding to the resulting system of algebraic equations will be called natural [11,12].

A feature of the construction of splines is the choice of slopes S_i'' so that the spline on the interval with the number n, respectively, becomes a continuation of the spline specified on the interval with the number $n-1$. To do this, it suffices to require the coincidence of the values of the third derivatives at the nodes x_l and x_{n-1}:

$$S_1^{(3)}(x_1)=S_2^{(3)}(x_1),\ \ S_{n-1}^{(3)}(x_{n-1})=S_n^{(3)}(x_{n-1}) \quad (5)$$

Below is a method for smoothing a dynamic process based on the cubic interpolation spline $S(x)$ on the $[a,b]$ interval.

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Method for calculating the coefficients of the interpolation spline. Let's assume that the grid $\Delta: a = x_0 < x_1 < K < x_n = b$ and some set of real numbers $f(x_i) = f_i$, $i = 0,1,K,n$ are given. Let us introduce the notation for the second derivatives of the spline at the nodes $S''(x_i) = m_i$.

On segment $[x_i, x_{i+1}]$, due to the linearity of the second derivative of the spline $[x_i, x_{i+1}]$, we have:

$$S''(x) = S''(x_i) + \frac{S(x_{i+1}) - S(x_i)}{x_{i+1} - x_i}(x - x_i), \text{ or}$$

$$S''(x) = m_i + \frac{\Delta m_i}{h}(x - x_i),\ \Delta m_i = m_{i+1} - m_i. \ (6)$$

Integrating (6) in the range from x_i to x, we get for $S'(x)$:

$$S'(x) = S'(x_i) + m_i(x - x_i) + \frac{\Delta m_i}{2h}(x - x_i)^2, \ (7)$$

Repeated integration in the same limits gives the expression for $S(x)$:

$$S(x) = f_i + S'(x_i)(x - x_i) + \frac{m_i}{2}(x - x_i)^2 + \frac{\Delta m_i}{6}(x - x_i)^3. \ (8)$$

Assuming in equality (8) $x = x_{i+1}$, we find

$$\Delta f_i = f_{i+1} - f_i = S'(x_i)h + \frac{m_i}{2}h^2 + \frac{\Delta m_i}{6}h^3,$$

Whence it follows that

$$S'(x_i) = \frac{\Delta f_i}{h} - \frac{h}{6}(2m_i + m_{i+1}), \ (9)$$

$$S(x) = f_i + \left[\frac{\Delta f_i}{h} - \frac{h}{6}(2m_i + m_{i+1})\right](x - x_i) + \frac{m_i}{2}(x - x_i)^2 + \frac{\Delta m_i}{6h}(x - x_i)^3. \ (10)$$

Substituting now the expression for $S(x_i)$ from (10) into (7), on the segment $[x_i, x_{i+1}]$ we get:

$$S'(x) = \frac{\Delta f_i}{h} - \frac{h}{6}(2m_i - m_{i+1}) + m_i(x - x_i) + \frac{\Delta m_i}{2h}(x - x_i)^2. \ (11)$$

Similarly, on segment $[x_i, x_{i+1}]$ we have

$$S'(x) = \frac{\Delta f_i}{h} - \frac{h}{6}(2m_{i-1} + m_i) + m_{i-1}(x - x_i) + \frac{\Delta m_i}{2h}(x - x_i)^2. \ (12)$$

So, as $f(x) \in C^{(2)}[a,b]$, then $S'(x_{i+0}) = S'(x_{i-0})$ from relations (11) and (12) follows:

$$\frac{\Delta f_i}{h} - \frac{h}{6}(2m_i + m_{i+1}) = \frac{\Delta f_{i-1}}{h} - \frac{h}{6}(2m_{i-1} + m_i) + hm_{i-1} + \frac{h}{2}\Delta m_{i-1}.$$

Let's rewrite this expression in a different form:

$$m_{i-1} + 4m_i + m_{i+1} = f_\Delta(x_{i-1}, x_i, x_{i+1});\ i = 1,2,K,n-1, \quad (13)$$

Where $f_\Delta(x_{i-1}, x_i, x_{i+1})$ is the second divided difference of the $f(x)$ function?

Thus, the equations for the internal nodes of the grid $x_i\,(i = 1,2,K,n)$ have the recurrent form

$$\begin{cases} m_0 + 4m_1 + m_2 = 12 f_\Delta(x_0, x_1, x_2); \\ m_1 + 4m_2 + m_3 = 12 f_\Delta(x_1, x_2, x_3); \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ m_{n-2} + 4m_{n-1} + m_n = 12 f_\Delta(x_{n-2}, x_{n-1}, x_n); \end{cases} \quad (15)$$

The system of equations corresponding to the interpolation cubic non-periodic spline $S(x)$ has a matrix of order $(n+1)\cdot(n+1)$:

$$A = \begin{pmatrix} 4 & \alpha & 0 \dots & 0 & 0 & 0 \\ 1 & 4 & 1 \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 \dots & 1 & 4 & 1 \\ 0 & 0 & 0 \dots & 0 & \beta & 4 \end{pmatrix} \quad (16)$$

If for the interpolated function the values of its derivative at the ends of the interval AA are known and equal to BB, then the boundary conditions are given in the form:

$$\alpha = \beta = 2,$$

$$c_\alpha = \frac{12}{h}\left(\frac{f_1 - f_0}{h} - f'_0\right), \quad c_\beta = \frac{12}{h}\left(f'_n - \frac{f_n - f_{n-1}}{h}\right)$$

Algorithm for calculating the coefficients of the equation. To solve systems of algebraic equations with matrices characterized by diagonal dominance, a general algorithm has been developed in accordance with the sweep method [13,14]. It is as follows.

Let the system of algebraic equations have the form $A_g = f$, where

$$A = \begin{pmatrix} d_1 & a_1 & 0 \dots & 0 & 0 & 0 \\ c_2 & d_2 & a_2 \dots & 0 & 0 & 0 \\ 0 & c_3 & d_3 \dots & 0 & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 \dots & c_{n-1} & d_{n-1} & a_{n-1} \\ 0 & 0 & 0 \dots & 0 & c_n & dn \end{pmatrix}$$

$f = \{f_1, f_2, K, f_n\}$ - given vector.

Solution $g = \{g_1, g_2, K, g_n\}$ is sought in the form of a recurrent procedure

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$g_i = \alpha_i g_{i+1} + \beta_i, \quad i = 1,2,K,n-1.$ (17)

From the i -th equation of the system we have:

$c_i g_{i-1} + d_i g_i + a_i g_{i+1} = f_i$ (18)

Let us exclude the unknown g_{i-1} from equations (17) and (18), replacing i within the last one. After elementary transformations, we obtain

$$(d_i + c_i\alpha_{i-1})g_i + a_i g_{i+1} = f_i - c_i\beta_{i-1} \quad (19)$$

Comparing this relation with (17), we obtain recursive formulas for the α_i, β_i coefficients (forward sweep):

$$\alpha_i = -\frac{a_i}{d_i + c_i\alpha_{i-1}}; \quad (20)$$

$$\beta_i = \frac{f_i - c_i\beta_i}{d_i + c_i\beta_i}; \qquad i = 1,2,K,n. \quad (21)$$

It is obvious that $g_n = \beta_n$. All other unknowns are found by formulas (17) (reverse sweep).

Conclusion. The proposed computational structure based on the basic cubic spline makes it possible to save two times the amount of memory compared to existing systems, and also to develop a software package for simulating signal processing processes using spline function methods that allow processing data of a continuous nature used in various fields of research.

It has been established that the use of the proposed spline functions as an apparatus for approximating functions in numerical analysis makes it possible in all cases to achieve tangible results in comparison with the classical apparatus of polynomials. In some problems, the transition to splines leads to an increase in the accuracy of the results, in others - to a significant reduction in computational costs, in others - both effects are achieved simultaneously.

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