IJDPP
International Journal of Development and Public Policy
| e-ISSN: 2792-3991 | www.openaccessjournals.eu | Volume: 1 Issue: 7

Applications of Differential Equations

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Abstract
The use and solution of differential equations is an important field of mathematics. Many physical laws governing certain phenomena are written in the form of a mathematical equation expressing a certain relationship between some quantities.

Keywords: physical laws, mathematical, language.

Let's try to explain it in a more understandable language. So, often we are talking about the relationship between the values that change over time, for example, the economy of the engine, measured by the distance that a car can travel on one liter of fuel, depends on the speed of the car.

For example, observational evidence suggests that the temperature of a cup of tea (or some other liquid) in a room of constant temperature will cool over time at a rate proportional to the difference between the room temperature and the temperature of the tea.

In symbols, if t is the time, M is the room temperature, and $f(t)$ is the temperature of the tea at time t then

$$f'(t) = k \cdot (M - f(t))$$

where $k > 0$ is a constant which will depend on the kind of tea (or more generally the kind of liquid) but not on the room temperature or the temperature of the tea. This is Newton’s law of cooling and the equation that we just wrote down is an example of a differential equation. Ideally we would like to solve this equation, namely, find the function $f(t)$ that describes the temperature over time, though this often turns out to be impossible, in which case various approximation techniques must be used.

Informally, a differential equation is an equation in which one or more of the derivatives of some function appear. Typically, a scientific theory will produce a differential equation (or a system of differential equations) that describes or governs some physical process, but the theory will not produce the desired function or functions directly.

The rate at which distance changes over time is determined by speed; therefore, speed is a derivative of distance; similarly, acceleration is a derivative of speed, since acceleration sets the rate at which speed changes over time. The great importance that differential equations have for mathematics, and especially for its applications, are explained by the fact that the study of many physical and technical problems is reduced to the solution of such equations. Differential equations play an essential role in other sciences as well, such as biology, economics and electrical engineering, military science; in fact, they arise wherever there is a need for a quantitative (numerical) description of phenomena (as soon as the surrounding world changes in time, and conditions change from one place to another) [1].

Let's look at a few examples:

Example.1. When administered intravenously using a dropper, the rate of glucose entering the blood is constant and equal to ***s***. In the blood, glucose is broken down and removed from the circulatory system at a rate proportional to the amount of glucose available. Then the differential equation describing this process has the form:

$$\frac{dx}{dt} = C - \alpha x$$

where x is the amount of glucose in the blood at the current time; C - the rate of glucose entering the blood; α is a positive constant. We write this equation in the form:

$$x' + \alpha x = C$$

Example.2. State Foreign Minister Blefuscu (for example!) argues that the weapons program adopted by Lilliputia (for example!) is forcing his country to increase military spending as much as possible. Lilliputian Foreign Minister also makes similar statements. The resulting situation (in its simplest interpretation) can be accurately described by two differential equations. Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be the cost of arming Lilliputia and Blefuscu. Assuming that Lilliputia increases her armament spending at a rate proportional to the rate at which Blefusc's armament spending increases, and vice versa, we get:

$$\frac{dx}{dt} = k\frac{dy}{dt} - \alpha x\ ,$$

$$\frac{dy}{dt} = l\ \frac{dx}{dt} - by$$

where the αx and $b\,y$ terms describe the military spending of each country, k and l are positive constants (This problem was first formulated in this way in 1939 by L. Richardson) [2].

Example.3. Consider the differential equation

$$y' = k \cdot y$$

When $k > 0$, this describes certain simple cases of population growth: it says that the change in the population y is proportional to the population. The underlying assumption is that each organism in the current population reproduces at a fixed rate, so the larger the population the more new organisms are produced. While this is too simple to model most real populations, it is useful in some cases over a limited time. When $k < 0$, the differential equation describes a quantity that decreases in proportion to the current value; this can be used to model radioactive decay.

After the problem is written in the language of differential equations, one should try to solve them, i.e. find the quantities whose rates of change are included in the equations.

Sometimes solutions are found in the form of explicit formulas, but more often they can be presented only in an approximate form or qualitative information about them can be obtained. It is often difficult to ascertain if a solution exists at all, let alone find one. An important section of the

ISSN 2792-3991 (online), Published under Volume: 1 Issue: 7 in December -2021

theory of differential equations is constituted by the so-called "existence theorems", in which the existence of a solution for one or another type of differential equations is proved.

A first order differential equation is an equation of the form

$$F(t, y, y') = 0$$

A solution of a first order differential equation is a function $f(t)$ that makes

$$F(t, f(t), f'(t)) = 0$$

for every value of t.

Here, F is a function of three variables which we label t, y, and y'. It is understood that y' will explicitly appear in the equation although t and y need not. The term “first order" means that the first derivative of y appears, but no higher order derivatives do.

A first order initial value problem is a system of equations of the form

$$F(t, y, y') = 0, \quad y(t_0) = y_0$$

Here t_0 is a fixed time and y_0 is a number. A solution of an initial value problem is a solution $f(t)$ of the differential equation that also satisfies the initial condition $f(t_0) = y_0$.

The general first order equation is rather too general, that is, we can’t describe methods that will work on them all, or even a large portion of them. We can make progress with specific kinds of first order differential equations. For example, much can be said about equations of the form

$$y' = \phi(t, y)$$

where ϕ is a function of the two variables t and y. Under reasonable conditions on ϕ, such an equation has a solution and the corresponding initial value problem has a unique solution. However, in general, these equations can be very difficult or impossible to solve explicitly [3].

Also as we have seen so far, a differential equation typically has an infinite number of solutions. Ideally, but certainly not always, a corresponding initial value problem will have just one solution. A solution in which there are no unknown constants remaining is called a particular solution.

REFERENCE

1. K.F.Riley, M.P.Hobson, S.J.Bence. Mathematical methods for physics and engineering. – Cambridge, 2006.
2. Применение дифференциальных уравнений в моделировании экономических процессов. Кукленкова А.А. ФГБОУ ВО «Елецкий государственный университет им. И.А. Бунина».
3. Claudio Canuto, Anita Tabacco. Mathematical Analysis I, II. Springer-Verlag Italia, Milan 2015.

ISSN 2792-3991 (online), Published under Volume: 1 Issue: 7 in December -2021