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Radial-Critical Graffs with J=3 Verses and Cyclomatic Number λ= 2

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Abstract: In this paper, we study ordinary radial-critical graphs without loops and multiple edges, with a cyclomatic number λ=2 and with j=3 hanging vertexes.

Keywords: Graph, vertex, edge, radius, pendant vertex, peripheral vertex, central block.

Let an ordinary graph be given $L=(X,U)$, where X- set of vertices and U - set of edges of this graph. Then the number $\lambda=m-n+1\geq 0$ is called the cyclomatic number of the graph L , where m-number of edges and n- is the number of vertices in a given graph. This means that there are λ edges such that, once they are removed, the resulting graph becomes a connected tree. A vertex x of a graph is called hanging if its degree is $s(x)=1$. The distance between the vertices x and y is denoted by $\rho(x,y)$. Vertices x_1 and x_2 are called similar if $\{x\in X\setminus\{x_1\}/\rho(x_1,x)=1\}=\{x\in X\setminus\{x_2\}/\rho(x_2,x)=1\}$.

The diameter of the graph L is $d(L)=\max_{x,y\in X}\rho(x,y)$. The radius of a graph is $r=\min_{x\in X}\left(\max_{y\in X}\rho(x,y)\right)$. A graph is called radial-critical if, after adding any new edge, its radius decreases.

It has been proved by the author earlier ([1]) that by extending non-peripheral vertices of radial-critical graphs without radial-critical similar vertices one can obtain radial-critical graphs.

Lemma 1. If in a radial-critical graph L a vertex x_0 is peripheral, then there exists a central vertex z_0- such that

$$\rho(z_0,x_0)=r,\ |x\in L\setminus \max_{x\in L}\rho(z_0,x)=r|=1.$$

Lemma 2. If in a radial-critical graph L a vertex x0 is peripheral, then any central vertex z_0 of graph L_{+u}, where $u=yy'$, satisfying conditions $\rho(z_0,x_0)=\rho(z_0,y)+\rho(y,y')+\rho(y',x_0)=r$, $\rho(z_0,y)=r-3$, $\rho(y,y')=2$, $\rho(y',x_0)=1$, is also central to the original graph L.

Lemma 3. If in a radial-critical graph L for vertex y_0 we have $|y\in L\setminus \max_{x\in L}\rho(y_0,x)=r|>1$, where y is hanging, then vertex y_0 is not central to L_{+u}, where $u=yy'$, $\rho(y_0,y')=\rho(y_0,y)=r-2$, $\rho(y,y')=2$.

We denote by G(j, λ) the class of radial-critical graphs with j hanging vertices and cyclomatic number λ. The class G(j,1) is described in [2].

Approval 1. Class G(0,2) - empty.

Proof. If $l(P_1\cup P_3)\leq 2r-1$, then the radius of the graph will be smaller than r, because $\max_x\rho(y_1,x)\leq r-1$.Consequently , $l(P_1\cup P_3)\geq 2r$. Then in the initial graph at $l(P_2)>2$ и $l(P_3)>1$ after adding an edge $u=x_2x_3$, where $x_2\in P_2$, $x_3\in P_3$, $\rho(x_2,y_2)=\rho(x_3,y_2)=1$ or adding a edge $u'=x'_2x'_3$, где $x'_2\in P_2$, $x'_3\in P_3$, $\rho(x'_2,y_1)=\rho(x'_3,y_1)=1$ to reduce the radius of the resulting graph would require the existence of at least two completely different central vertices. Let these central vertices z_0 and z'_0. Then because

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of the fact that $l_2 \leq l_3$, the furthest points from these central vertices are in the chain P_3. In that case, it would be $l(P_1 \cup P_2) < r$-1, which is impossible, because then there would be a vertex $\bar{z} \in P_1 \cup P_2$ such that $\max_x \rho(\bar{z}, x) = r - 1$.

If $l(P_3)$=1, then instead of point x_3 it is possible to take a point y_1, and instead of a point x'_3 it is possible to take a point y_2 – the result will be exactly the same.

In the case of $l(P_2)$=2 and $l(P_3)$=1, should be $l(P_1 \cup P_2) \leq 2r - 1$. Otherwise, adding an edge $u=x_2z_0$, where $\rho(x_2,z_0)=2$, $z_0 \in P_1$, (or edges $u=x'_2z'_0$, where $\rho(x'_2,z'_0)=2$, $z'_0 \in P_1$) does not reduce the radius of the graph, since in both cases

$$\forall x_0 \in L\ \exists x'_0 \in P_1 \cup P_3\ [\rho_{G+u}(x_0, x'_0) \geq r],$$

which contradicts the criticality of the graph. However, in this case we have some vertex $t_0 \in P_1$, for which $\max_{x \in L} \rho(t_0,x) < r$. Consequently, the original graph is not radial-critical and the class $G(0, 2)$-emoty.

Approval 2. Class $G(1,2)$ – empty.

Proof. Then in such a graph there is only one articulation point on which the suspended chain hangs ([3]). Let y_3- articulation point, and P_0 - suspended chain length k, a $\bar{x}$ - hanging apex of this chain.

1. $y_3 \in P_1 \backslash \{y_1,y_2\}$. Due to the fact that the diameter of the graph does not exceed $2r$-2, $\exists x \in P_1 \cup P_2 \cup P_3 [\max_x \rho(\bar{x}, x) \leq 2r - 2]$ и $l(P_1 \cup P_2) \leq 2r - k$. If here, $l(P_1 \cup P_2) = l(P_1) + l(P_2) = l_1 + l_2 \geq 2r - k$, then adding an edge $u=y'_3y''_3$, where $y'_3 \in P_1$, $y''_3 \in P_0$, $\rho(y''_3,y_3) = \rho(y_3,y'_3)=1$, does not reduce the radius of the graph, since for any central vertex z_0 of the original graph $\exists x \in P_2[\rho_{G+u}(z_0, x) \geq r]$, which is impossible due to the criticality of the graph.Consequently , $l_1+l_2 \leq 2r - k - 1$. Suppose that $l_2 = l_3$. It is obvious that $l_2 = l_3 \geq 2$. Then adding an edge $u=y'_1 y''_1$, where $\rho(y'_1,y''_1)=2$, $\rho(y''_1,y_1) = \rho(y_1,y'_1)=1$, $y'_1 \in P_2$, $y''_1 \in P_3$ does not reduce the radius of the graph, since $l_2+l_3 < 2r - k - 1$. Consequently, $l_2 > l_3$.

We prove that $\rho(y_3,y_1) = \rho(y_3,y_2)$. Indeed, if $l_1 \geq 5$, then adding edges of the form $u=y'_3y''_3$ or $u=\bar{y}'_3y''_3$, where $y'_3 \in P_1$, $\bar{y}'_3 \in P_1$, $\rho(y_3,\bar{y}'_3)=1$, $\rho(y'_3,\bar{y}'_3)=2$, shows that there exist vertices z_0 and z'_0 – central, for which

$\rho(z_0,\bar{x}) = \rho(z'_0,\bar{x}) = r$. Then $\exists \bar{x}' \in P_2[\rho(z_0,\bar{x}') = \max_{x \in Q} \rho(z_0,x) = r-1]$ and $\exists \bar{x}'' \in P_2[\rho(z'_0,\bar{x}'') = \max_{x \in Q} \rho(z'_0,x) = r-1]$, where $Q = P_2 \cup P_3$.

Therefore $l_1+l_2=2r$, otherwise adding an edge $u=y'_3\bar{y}'_3$ does not reduce the radius of the graph. In that case, all vertices from z_0 to z'_0 in chain P_1 will be central (odd number and at least five vertices). It follows that l_1 –even number. Note that , что $l_1<5$ is impossible.

Let l_3=1. Let's add an edge to the original graph. $u=y_2y'_2$, where $\rho(y_1,y'_2)=2$, $\rho(y_2,y'_2)=1$, $y'_2 \in P_2$ or $u=y_2y'_1$,where $\rho(y_2,y'_1)=2$, $\rho(y_1,y'_1)=1$, $y'_1 \in P_2$. It can then be seen that l_2+l_3 –even , and l_2 –

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odd. Then adding an edge $u=y_3y'_3$ does not reduce the radius of the graph. Consequently, $l_3\geq 2$. Since l_1 –even and $l_1+l_2=2r$,

$l_2$wil ne also even; therefore $l_3$will be also even. Now, if $l_2= l_3+2$, then adding an edge of type $u=y''_1\bar{y}'_1$, where $\rho(y'_1,\bar{y}'_1)=1$, $\rho(y_1,\bar{y}'_1)=2$, $\bar{y}'_1\in P_2$, does not reduce the radius of the graph. Therefore, under these conditions we would have $l_1\geq l_2\geq l_3+4$, which is impossible. 2. $y_3\in P_2\setminus\{y_1, y_2\}$. Similarly, as in point 1, it is proved that $(\rho(y_1,y_3)= \rho(y_3,y_2)$ and l_1, l_2, l_3 are even, $l_1\geq l_2\geq l_3+2$. It is known that if $l_2=4$ we would have $l_1=6,8,10$ and etc.. Therefore, this case is also impossible.

3. $y_3\in P_3\setminus\{y_1, y_2\}$. This case is also not possible because adding an edge $u=y'_1y''_1$(or $u= y'_2y''_2$) does not reduce the radius of the graph.

4. $y_3\in\{y_1, y_2\}$. Without detracting from the generality, it can be assumed that $y_3=y_1$. Suppose that after adding an edge $u=c_1y'_1$, where $c_1\in P_0$, $y'_1\in P_1$, $\rho(c_1,y'_1)=2$, the radius of the graph is reduced by one unit. In the resulting graph the central vertex is either a single vertex $z_1\in P_1$, where $\rho_{G+u}(z_1,\bar{x}) = r-1$, either the vertex $\bar{z}$ from $P_2\cup P_3$, where $\rho_{G+u}(\bar{z},\bar{x}) = r-1$. Then adding an edge $u=x_2x_3$, where $x_2\in P_2$, $x_3\in P_3$, $\rho(x_2,x_3) = 2$, $\rho(x_2, y_3) = \rho(y_3, x_3) = 1$, must reduce the radius of the original graph by one. In the resulting graph $G+u$ the central vertex will be $\bar{z}'$, for which$\rho(\bar{z}', y_2)+ \rho(y_2, x_3)+1+ \rho(x_2,x'_2)=r-1$, where $x'_2$is the point farthest from in the original graph. Consequently, it would also be central to the original graph. In this case, max $(l(P_1\cup P_3), l(P_1\cup P_2))=r$, otherwise adding edges $u=y'_1x_2$ and $u= y'_1x_3$ does not reduce the radius of the graph, which is impossible due to the criticality of the graph. It follows that this case is also impossible.

Consequently, the class $G(1, 2)$ – empty.

About class $G(2, 2)$ look at [4].

Let the central block of graph L consist of three simple chains P_1, P_2 and P_3 with common ends y_1 and y_2 (they do not intersect at other points), where $l(P_1)\geq l(P_2)\geq l(P_3)$, y_3, y_4 and y_5 are points of joint, on simple chains, and Π_3, Π_4 and Π_5 are simple chains suspended from these points.

Lemma 4. If $l_3=1$, then $l_2\leq 3$.

Proof. Suppose the contrary, that $l_2\geq 4$. Then, after adding an edge $u=y_{01}y_{02}$, where y_{01}, $y_{02}\in P_2$, $\rho(y_1,y_{01})=\rho(y_2,y_{02})=1$, $\rho(y_1,y_{02})= \rho(y_1,y_2)+ \rho(y_2,y_{02})=2$, $\rho(y_2,y_{01})= \rho(y_2,y_1)+ \rho(y_1,y_{01})=2$, we have $\forall\bar{x}\in P_1$, $x\in P_2[\rho_L(\bar{x}, x)= \rho_{L+u}(\bar{x}, x)$, i.e. the radius of the resulting graph does not change, which contradicts the criticality of the original graph. Consequently, $l_2\leq 3$.

Lemma 5. If $l_3=1$, $l_2\leq 2$, then vertices y_1 and y_2 cannot be joint points.

Proof. Suppose the contrary, i.e. let y_1 be the articulation point and a chain Π_3 of length $k\leq r-2$ suspended from this vertex. Then add an edge to the graph L $u=\bar{x}\bar{y}$, where $\bar{x}\in P_2\setminus\{y_1, y_2\}$, $\bar{y}\in\Pi_1$, $\rho(\bar{x},\bar{y})=2$. If the radius of the graph now decreases, then the centre of the graph L_{+u}, will be some vertex $c\in P_1$, for which $\max_{x\in B}\rho(c,x)=r-1$ where B- central block. Here $\rho(c, \overline{y_1})=$ $= \rho(c, y_2)+\rho(y_2, y_1)+\rho(y_1, \overline{y_1})=\rho(c, y_2)+2$, $\rho_{L+u}(c,\bar{y})= \rho_{L+u}(c, y_2)+ \rho_{L+u}(y_2,\bar{x})+ + \rho_{L+u}(\bar{x},\bar{y})= \rho(c,$

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y_2)+2, i.e. adding an edge does not affect the distance from the vertex c to the vertex $\overline{y_1}$, , which contradicts the criticality of the original graph.

Consequently, vertex y_1 cannot be an articulation point.

Lemma 6. If l_3=1, then l_2=2.

Proof. Assume the opposite, i.e l_2= 3.Given that $r(B)\leq r$, we have $l_1=2r-3$, or $l_1=2r-4$. If $l_1=2r-4$, then adding an edge $u=y_1y_0'$ (where $\rho(y_1,y_0')=2$, $\rho(y_2, y_0')=1$]does not decrease the radius of the resulting graph. Therefore, only a case of $l_1=2r-3$. If $\rho(y_1,z_1)=\rho(y_2,z_2)=r-3$, $\rho(z_1,z_{10})=\rho(y_2,z_{20})=\rho(z_{10},z_{20})=1$, then $\rho(y_1,z_{10})=\rho(y_2,z_{20})=r-2$. Now, if $\rho(z_{10},\bar{x})=\rho(z_{10}, z_2)+\rho(z_2,\bar{x})=2+(r-2)=r$, where $\bar{x}$ – pendant vertex, then after adding an edge $u=z_{10}z_2$ (where $\rho(z_{10}, z_2)=2$) the radius of the graph is reduced by one. In this case, after adding edge $u=z_{10}\overline{z_1}$ (where $\rho(\overline{z_1}, z_1)=$

$=\rho(\overline{z_1}, z_{10})=1$, $\rho(\overline{z_1}, y_1)=r-4$) the radius of the graph does not decrease, which contradicts the criticality of the original graph.

The following theorem follows from these lemmas.

Theorem. For any $r>3$ and $d=2r-2$ there exists a radial-critical graph $L\in G(3,2)$ with $n=3r-1$ vertices (three of which are pendent) and $m=3r$ edges (cm. Fig 1.).

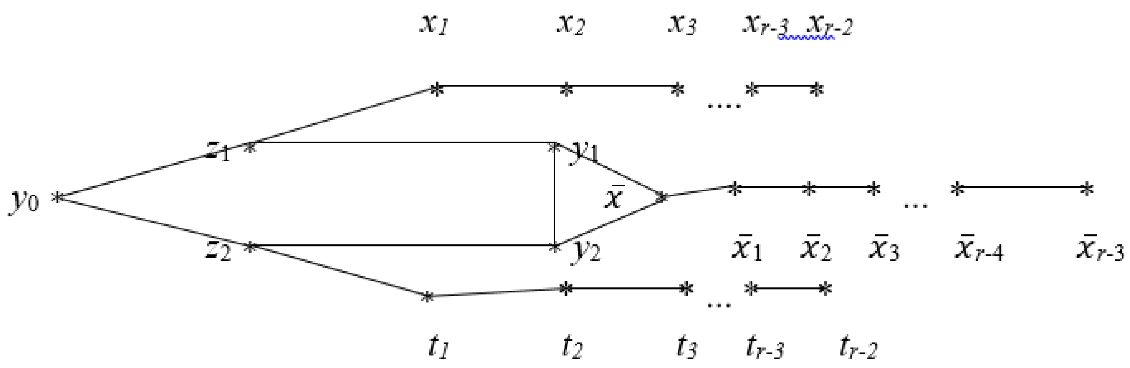

Рис. 1

Reference

1. Nishanov Y. Radial-critical graphs with maximum diameter. " Computational and applied mathematics issues ", 1972.,edition. 15, Tashkent.

2. Nishanov Y. Monocyclic radial-critical graphs. "Problems of algebra and number theory",Samarkand: Edition. SamSU, 1990., 54-65.

3. Zikov A.A. Fundamentals of graph theory. – M.: "Science", 1987., p.384.

4. Yusuf Nishanov. Radially critical graphs with given cyclomatic number and pendant vertices. "International Journal for Innovative Engineering and Management Research" (ISSN 2456-5083), Vol-09, Issue-12 Dec-2020, 173-177.

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