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Peculiarities of Dirichlet Character

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Annotation: Historians commonly take the "modern" age of mathematics to have begun in the nineteenth century. But although there is consensus that the events of that century had a transformational effect on mathematical thought, it is not easy to sum up exactly what changed, and why. Aspects of the transformation include an increasingly abstract view of mathematical objects; the rise of algebraic methods; the unification of disparate branches of the subject; evolving standards of rigor in argumentation; a newfound boldness in dealing with the infinite; emphasis on "conceptual" understanding, and a concomitant deemphasis of calculation; the use of (informal) set-theoretic language and methods; and concerns to identify a foundational basis to support the new developments.1 It is still an important historical and philosophical task to better understand these components, and the complex interactions between them.

Keywords: geometrical content, integral logic problems, mathematical analysis, "pigeonhole principle"

The variety of problems is great, and the number of ways of solving them is also great. When solving many problems, one may encounter a method of reasoning - "the other way around". This method of problem solving is called the Dirichlet principle. A rather simple formulation of the Dirichlet principle gives the possibility to solve logic problems, problems of geometrical content.

The so-called "Dirichlet principle", named after the German mathematician Peter Gustav Lejeune Dirichlet (1805 - 1859), also called the "box principle" or the "pigeonhole principle", is often useful in solving a variety of problems. This principle is often a good aid in the proof of the most important theorems in number theory, algebra and geometry.

Made a number of important discoveries in the theory of numbers, defined formulas for a number of classes of binary quadratic forms with a given determinant and proved the theorem on infinite number of prime numbers in the arithmetic progression of integers, the first term and the difference of which are mutually prime. To solution of these problems he applied analytical functions called Dirichlet functions (series). Created general theory of algebra, units in algebraic number field. In the field of mathematical analysis, he was the first to precisely formulate and investigate the concept of conditional convergence of series, he gave a rigorous proof of the possibility of Fourier expansion of piecewise continuous and monotone functions, which served as a basis for many future studies. Dirichlet's work in mechanics and mathematical physics, in particular in potential theory, is considerable. Dirichlet is associated with the problem, the integral (he introduced the integral with the Dirichlet kernel), the principle, the character, and the series. Dirichlet's lectures had a great influence on later distinguished mathematicians, including H. Riemann, F. Eisenstein, L. Kronecker and J. Dedekind.

Traditionally, the Dirichlet principle is for some reason always explained using the example of rabbits in cages: if the total number of rabbits is greater than the number of cages, there must be more than one rabbit sitting in one of these cages.

Also, this principle can look like this: it is impossible to have n+1 rabbits in n cells, i.e. there will be a cell with at least two rabbits sitting in it.

This principle can be formulated in terms of mappings between sets: If we map a set P containing n+1 element to a set Q containing n elements, then we will find two elements of the set P having the same image.

So, to apply Dirichlet's principle to problems, you need to specify what to take as "cells" and what to take as "rabbits", and also specify the way to put "rabbits" into "cells". In terms of mappings between sets this means that one has to specify not only sets P and Q, but also specify mapping between them.

Most often in problems not the Dirichlet principle is applied, but some property of it which is called the generalized Dirichlet principle.

Let us consider problems with application not of the Dirichlet principle but of a generalization of it which is formulated below and which is usually encountered in problems.

Generalization of Dirichlet's principle: given n cells and nk + 1 rabbits are placed in these cells. Then we will find a cell with at least k + 1 rabbits.

Problem 1. There are 29 students in the class. Sasha Ivanov made 13 mistakes in the dictation, and no one else made more mistakes. Prove that at least three students made the same number of mistakes.

Solution. Let's take as "cells" all possible variants of the number of mistakes. There are 14 of them, because pupils can make 0, 1, ..., 13 mistakes. And for "rabbits" we will take the pupils, who made the dictation. According to the condition there are 29 of them. Each of them will be put into a cell which corresponds to the number of mistakes they made. Then we will find the cell containing at least three rabbits, it means that there will be three pupils with the same number of mistakes.

Problem 2: There are 160 pupils in five classes of the school. Prove that there are 4 pupils having the same birthday.

Solution. There could be at most 53 weeks in a year. We take them as "cells" and take the children as "rabbits". We will place the "rabbits" on the "cells" corresponding to their birthdays. Because of Dirichlet's principle there will be at least four "cages", which means that there will be a week when four people have their birthday at once.

When solving problems using the Dirichlet principle one can do two things:

1) Assume the contradiction and calculate how many values are needed. Comparing with the given conditions, we arrive at a contradiction.

2) Choose what to take as "cells" and what to take as "rabbits". Applying the Dirichlet principle directly, we establish the existence of what we were looking for.

Also, with the help of Dirichlet's principle it is possible to solve problems in which we have to get some number of objects of different colours or types (for example: pairs of socks or gloves of different colours). These problems are given below.

Problem 3: A box contains 10 pairs of black gloves and 10 pairs of red gloves of the same size. How many gloves should be taken out of the box at random to be sure that among them there are:

a) Two gloves of the same colour;

b) One pair of gloves of the same colour;

Solution.

a) If we take the colours of the gloves as "squares", then taking any three seals we will get two gloves as "rabbits" in one of the "squares". This is what is required.

b) You can take 20 gloves for one hand and it is impossible to choose a single-colored pair of gloves from them, so the required number is at least 21. Prove that number 21 is the sought number.

Take as "cells" the colours of the gloves (there are two of them). Let's take the gloves as "rabbits". According to the generalized Dirichlet principle in one of the "cells" there will be at least 11 "rabbits". This means that there are 11 gloves of the same colour. But there are only 10 pairs of gloves of the same colour, so they cannot all be on the same hand. So, there is one pair of gloves of the same colour among those 11 gloves.

In spite of the perfect obviousness of this principle, its application is a very effective method of solving problems, giving in many cases the simplest and most elegant solution.

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The problems considered are among the simplest, but at the same time the main ones in the study of the Dirichlet principle. A sufficiently simple formulation of the Dirichlet principle makes it possible to solve problems related to number divisibility, probability theory, numerical sequences, arrangement in the plane of circles, polygons, logical problems, and geometrical problems.

Finally, we have argued that understanding and evaluating the considerations that shape mathematical language and inferential practice is an important part of the history and philosophy of mathematics. Over time, mathematics evolves in such a way as to support the pursuit of distinctly mathematical goals. One of these is the goal of maintaining an inferential practice with clear rules and norms, one that allows its practitioners to carry out, communicate, and evaluate arguments that can become exceedingly long and complex. Another is the goal of promoting efficiency of thought, leveraging whatever features we can to extend our cognitive reach and transcend our cognitive limitations.

Used literature:

1. Jeremy Avigad. Mathematical method and proof. Synthese, 153:105–159, 2006.
2. George Boole. A Treatise on Differential Equations. Macmillan and Co., Cambridge, 1859.
3. Harold M. Edwards. The genesis of ideal theory. Archive for History of Exact Sciences, 23:321–378, 1980.
4. Harold M. Edwards. Kronecker's algorithmic mathematics. Mathematical Intelligencer, 31:11–14, 2009.