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Computer Modeling of Two-Dimensional Non-Stationary Heat Conduction Problems Considering Point Heat Sources Using the FEM

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Abstract:

The process of heat exchange in mechanical systems is discussed in the article. The solution to two-dimensional non-stationary heat conduction problems in the presence of point heat sources is studied based on the finite element method. To solve a plane heat conduction problem, a triangular finite element with rectilinear sides is used. The basic relations for linear triangular finite elements are presented, taking into account the influence of point heat sources. A source set at a specific point on the plane describes a heater in the form of a string. This point on the plane is the trace of the string and is set by the heat generation power per unit length. Based on the developed computational algorithm and software, a numerical solution to a specific problem is given and the influence of point heat sources on the distribution of the temperature field of the structure is studied.

Keywords: heat transfer, heat source, convective heat transfer, non-stationarity, FEM.

In many engineering problems, knowledge of the temperature distribution in a structure is an important aspect. The amount of heat supplied to it or lost by it can be calculated provided that the temperature distribution is known. In the article, the temperature field is formed by considering the influence of a point heat source set at a specific point on the plane and describes the presence of a heater in the form of a string. The trace of the string is a point on the plane, determined by the power of heat generation.

Formulation of the problem

The problem of temperature distribution at different time points is solved in a plane formulation [1, 2]. To find the temperature field in the rectangular calculation domain (a plate) *ABCD* (Fig. 1), a two-dimensional non-stationary heat conduction problem is solved based on equation [4]:

$$\lambda \frac{\partial T}{\partial t} = K_{xx}\frac{\partial^2 T}{\partial x^2} + K_{yy}\frac{\partial^2 T}{\partial y^2} + Q, \qquad (1)$$

where $\lambda = c\rho$ is the specific volumetric heat capacity; c is the specific heat capacity of the material; ρ is the density; K_{xx}, K_{yy} are the heat conductivity coefficients in the corresponding directions; Q is the power of the heat source inside the body.

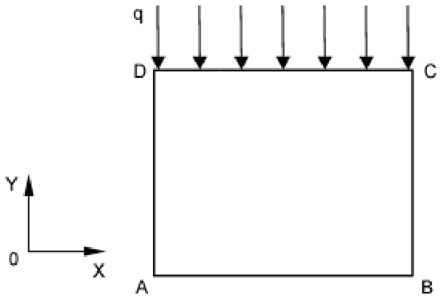

Fig. 1. Scheme of the computational domain

Symmetry conditions are set on lines *AD* and *BC* (the so-called natural boundary conditions [3]):

$$\left.\frac{\partial T}{\partial x}\right|_{AD} = 0, \quad \left.\frac{\partial T}{\partial x}\right|_{BC} = 0. \qquad (2)$$

A constant temperature (Dirichlet condition) is set on line *AB*, equal to the ambient temperature T_∞:

$$T|_{AB} = T_\infty. \qquad (3)$$

Conditions that determine the convective exchange of heat with the medium can be set on line *AB*:

$$K_{yy}\frac{\partial T}{\partial y} + h(T - T_\infty) = 0, \qquad (4)$$

where h is the heat transfer coefficient.

The temperature field throughout the entire computational domain at time t = 0: $T(x, y, 0) = T^0(x, y)$ is set as the initial condition.

In this case, the solution to the system consisting of equation (1) and boundary conditions (2) - (4) is reduced to minimizing the functional [4]:

$$\chi = \int_V \frac{1}{2}\left[K_{xx}\left(\frac{\partial T}{\partial x}\right)^2 + K_{yy}\left(\frac{\partial T}{\partial y}\right)^2 - 2QT + 2\lambda\frac{\partial T}{\partial t}T\right]dV + \int_{S_1}\frac{h}{2}(T - T_\infty)^2, \qquad (5)$$

where S_1 is the area of surface where convective heat exchange occurs.

Finite element solution to the problem

The temperature in the finite element is specified as the product of two independent functions:

$$T^{(e)} = N(x, y)T(t)$$

or in a matrix form:

$$T^{(e)} = \left[N_i(x, y)N_j(x, y)N_k(x, y)\right]\begin{Bmatrix} T_i(t) \\ T_j(t) \\ T_k(t) \end{Bmatrix}. \qquad (6)$$

For finite element e, the condition for the extremum of the functional leads to the following system of differential equations:

$$\left[c^e\right]\frac{\partial\{T\}}{\partial t}+\left[k^e\right]\{T\}+\left\{f^e\right\}=0, \quad (7)$$

here

$$[c^e]=\int_{V^e}\lambda[N][N]^T dV, \quad (8)$$

$$[k^e]=\int_{V^e}[B^e][D^e][B^e]^T dV+\int_{S_2}h[N][N]^T dS, \quad (9)$$

$$\{f^{(e)}\}=-\int_{V^{(e)}}Q[N^{(e)}]^T dV-\int_{S_2^{(e)}}h\varphi_\infty[N^{(e)}]^T dS, \quad (10)$$

where

V^e is the volume of the finite element;

$[N]$ is the shape function matrix;

$[B^e]$ is the matrix of shape function derivatives;

$[D^e]$ is the matrix of material properties containing thermal conductivity coefficients.

Integral (8) when solving a two-dimensional heat conduction problem has the following form (damping matrix):

$$[c^e]=\frac{\lambda A a_t}{12}\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix}, \quad (11)$$

where

A is the area of the finite element; a_t is the element thickness.

The thermal conductivity matrix (in the absence of convection) has the following form:

$$[k^e]=\frac{k_{xx}a_t}{4A}\begin{bmatrix}b_ib_i&b_ib_j&b_ib_k\\b_jb_i&b_jb_j&b_jb_k\\b_kb_i&b_kb_j&b_kb_k\end{bmatrix}+\frac{k_{yy}a_t}{4A}\begin{bmatrix}c_ic_i&c_ic_j&c_ic_k\\c_jc_i&c_jc_j&c_jc_k\\c_kc_i&c_kc_j&c_kc_k\end{bmatrix}, \quad (12)$$

where $b_i=y_j-y_k$; $c_i=x_j-x_k$; (the remaining quantities are obtained by circular permutation of indices *i*, *j*, *k*).

If side *i* of the *j* finite element is subject to convection, then the second integral in (9) is:

$$\int_{S_2}h[N][N]^T dS=\frac{h a_t L_{ij}}{6}\begin{bmatrix}2&1&0\\1&2&0\\0&0&0\end{bmatrix}, \quad (13)$$

where L_{ij} is the length of the side of the element between nodes *i* and *j*.

If heat is lost by convection between sides with nodes *j, k* or *k, i,* then matrix (13) is transformed [4].

We assume that the value of Q is constant inside the element, then:

$$Q\int_V [N]^T dV = Q\int_V \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} dV = \frac{QV}{3}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} . \qquad (14)$$

If convective heat exchange occurs on the side with nodes *i j*, then the "load vector" in solving the heat conduction problem is:

$$\int_{S_2} hT_\infty [N]^T dS = \frac{hT_\infty L_{ij} a_t}{2}\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}.$$

For a finite element mesh, a system of ordinary differential equations is written as:

$$[C]\frac{\partial\{T\}}{\partial t} + [K]\{T\} + \{F\} = 0, \qquad (15)$$

where

$$[C] = \sum_e [c^e]; \quad [K] = \sum_e [k^e]; \quad [F] = \sum_e [f^e].$$

Replacing the time derivative in equation (15) with its finite-difference analog, we obtain an implicit difference scheme for solving the heat equation using the finite element method [3-6]:

$$\left(\frac{[C]}{\Delta t} + [K]\right)\{T\}^{n+1} = \left(\frac{[C]}{\Delta t} - [K]\right)\{T\}^n - \{F\}^{n+1} \qquad (16)$$

Thus, if the temperature vector $\{T\}^n$ at time point t_n is known, then the temperature of the plate at time point $t_{n+1} = t_n + \Delta t$ is obtained as a result of solving the system of linear algebraic equations (16).

Computational experiment and analysis of results

The following problem is considered as a test example [7]:

Three cables pass through a heat-conducting medium, as shown in Fig. 2. The thermal conductivity coefficients of the medium is $K_x = K_y = 50\ \frac{W}{cm \cdot K}$. The heat transfer coefficient on the surface of the medium is $h = 10\ \frac{W}{cm^2 \cdot K}$. The medium under consideration is limited on the sides by a thick layer of insulation. Air temperature on the surface of the medium is $T_\infty = +40\ ^0C$. Temperature of the lower layer of the medium is $T_0 = +20\ ^0C$. The heat radiation power of each cable is $Q = 200W$. Copper has the following thermophysical characteristics (1, in Fig. 2): $\lambda =$ 384 $W/(m \cdot {}^oC), \rho =$ 8800 $kg/m^3, c =$ 381 $J/(kg \cdot {}^oC)$. Thermophysical parameters of the material of inclusion are: steel (2, in Fig. 2): $\lambda_2 = 46\,W/(m \cdot {}^\circ C), \rho_2 = 7800\,kg/m^3, c_2 = 460\,J/(kg \cdot {}^\circ C)$.

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Dimensions of the material of inclusion: n=18 cm, m=2.375 cm. Plate dimensions: a=5 cm, b=4 cm, c=5 cm, d=5 cm, e=4 cm.

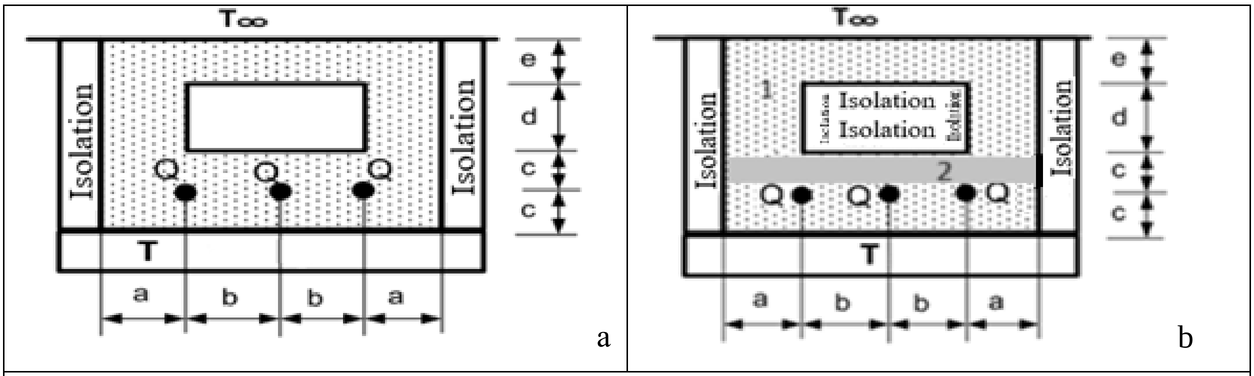
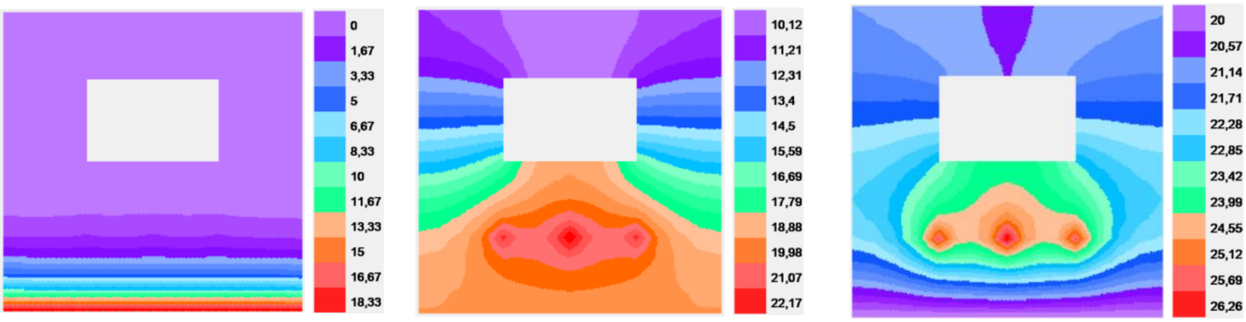


Fig. 2. Arrangement of cables in a heat-conducting medium

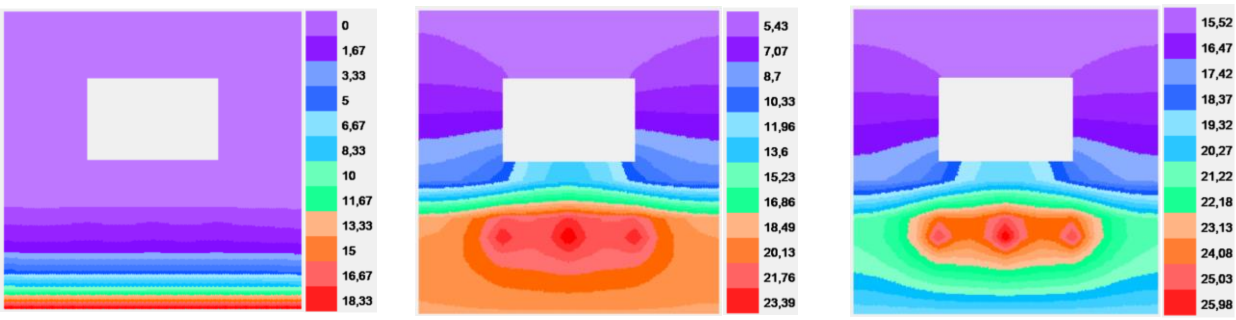
The temperature distribution in a homogeneous material (Fig. 2.a) at t = 6, 120, and 300 seconds is presented in Fig. 3. At the initial stage of the process (t = 6 sec), there is no influence of sources, then at (t = 120 and 300 sec), the influence of sources is observed, characterized by thermal concentration in the vicinity of sources [9-12].



t = 6 sec t = 120 sec t = 300 sec

Fig.3. Temperature distribution in a homogeneous material

In the presence of a rectangular steel area in a structure made of copper (Fig. 2 b), a redistribution of the generated heat is observed in its lower part (Fig. 4). The change in the temperature field reflects the difference in the thermal conductivity coefficients of copper and steel.



t =6 sec t =120 sec t =300 sec

Fig.4. Temperature distribution in a non-homogeneous material

For a specific point (2.5 cm, 11.5 cm) located on the left side of the cavity, table 1 presents temperature values that indicate the influence of the non-homogeneity of the structure, namely a decrease in temperature in the vicinity of sources.

Table 1. Temperature at point (2.5 cm, 11.5 cm)

Plate material	t=6 sec	t=120 sec	t=300 sec
Homogeneous	0.0358 ℃	17.6320 ℃	30.3977 ℃
Non-homogeneous	0.0016 ℃	10.5583 ℃	23.5279 ℃

Conclusion

Thus, the presence of sources increases the temperature field in the vicinity of sources, and the presence of non-homogeneity changes the overall temperature background of the structure.

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